

The Economic Relationship and Unit Roots in Chile*

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I Introduction

The purpose of this paper is to examine the economic relationship between consumption, production and consumption prices in Chile, using the data obtained from the Penn World Table. In order to explain how consumption level can be determined by production and/or consumption prices, the data of real GDP per capita (Laspeyres index: 1985 intl. prices) denoted by RGDPL, real consumption per capita (1985 intl. prices) by RC, and consumer prices (price level consumption: %), which are obtained by dividing the purchasing power parity of consumption by the exchange rate with US dollar, denoted by PC are used in this paper. In addition to the conventional regression analysis, the existence of unit roots among the above data will also be tested. This is because recent popular discussions in applied econometrics on long-run economic relationships are based on the existence of a unit root and also because the assumptions of the classical regression model necessitate that the sequences of economic

variables be stationary and also that the errors have a zero mean and finite variance. However, the existence of a unit root violates these assumptions, and in the presence of nonstationary variables there might be what Granger and Newbold (1974) call a *spurious regression*. A spurious regression has a high R^2 , t-statistics that appear to be quite significant, but the results are without any economic meaning.

To clarify the analysis and the steps undertaken, this paper is organised as follows: Section 2 roughly describes exploratory data analysis, which will enrich the following two sections, and Section 3 shows the estimation of dynamic model by the OLS method as well as its problems. Section 4 tests for the existence of unit roots and Section 5 consists of tests for several restrictions and the evaluation of the tests. Section 6 summarises and concludes this paper.

II Characterisation of the Relationship between Variables and their Dynamic Structure

In order to decide which variables in the data are appropriate for the following regression analysis, several transformation of the data should be examined before conducting the regression analysis.¹

As the purpose of this paper is to explain

*) This paper has been produced partly based on the project in which the author was involved at the Department of Economics, University of Essex while the author was visiting the university, whose hospitality and support are acknowledged with thanks. Views and errors are, of course, mine.

how consumption level can be determined by income and consumption prices in Chile, it will be reasonable to examine the time series data of these three variables, real consumption per capita series (RC), real GDP per capita series (RGDPL) and consumer prices series (PC)². Fig. 1 - Fig. 3 show these three time series data in level respectively (Fig. 4 - Fig. 6 in logarithm correspondingly), and Fig. 7 - Fig. 10 show the relationship of consumption with income and prices in both level and logarithm. These figures suggest that there will be the positively strong relationship between consumption and income and that consumption will be related

with consumption prices negatively. Fig. 9 and Fig. 10 hint that RGDPL in logarithm and PC in logarithm should be used in the following regression analysis.



Fig. 1 Real Consumption per capita (RC)

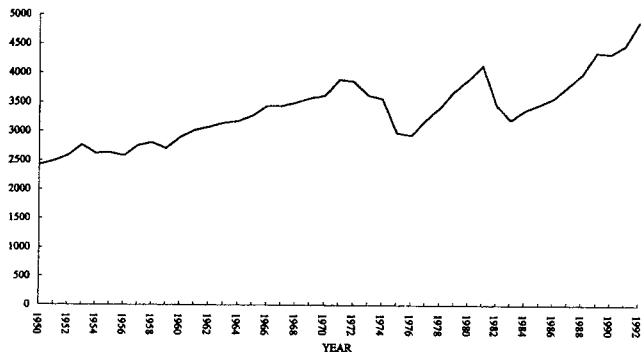


Fig. 2 Real GDP per capita (RGDPL)

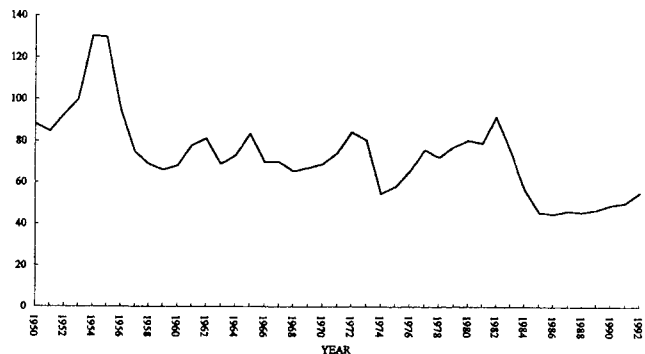


Fig. 3 Consumption Prices Level

1 All results reported in this paper have been calculated using the software, Microfit.

2 As to the definition of the prices used in this paper, two types of variables have been examined in addition to PC in order to obtain good estimation of parameters. Denoting two types of inflation for consumer prices by *PCINF1* and *PCINF2*, these two types are defined such that:

$$PCINF1_t = \frac{PC_t - PC_{t-1}}{PC_{t-1}}$$

$$PCINF2_t = \frac{PC_t}{PC_{t-1}}$$

The reason why these two types have not been used in this paper is that *PCINF1* can not be used for logarithm when it has negative value (in fact negative values of *PCINF1* have been calculated from the data of Chile), and that desirable results of estimation of parameters could not have been obtained using the second definition of inflation rate, *PCINF2*.

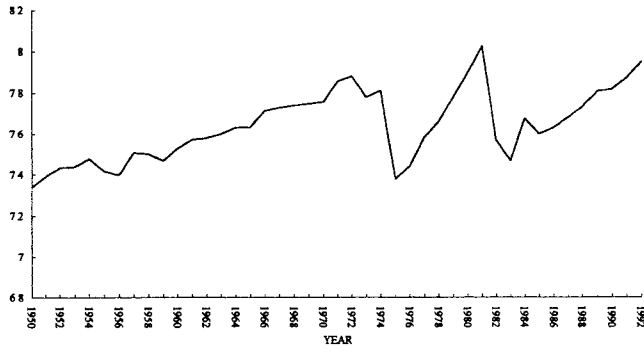


Fig. 4
Real Consumption per capita in logarithm (LNRC)

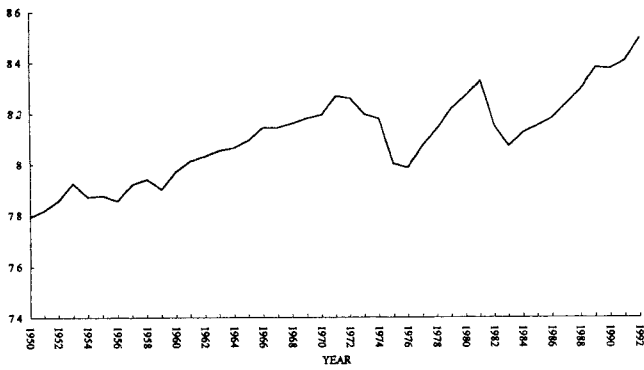


Fig. 5
Real GDP per in logarithm (LNRGDP)



Fig. 6
Consumption Prices Level in logarithm

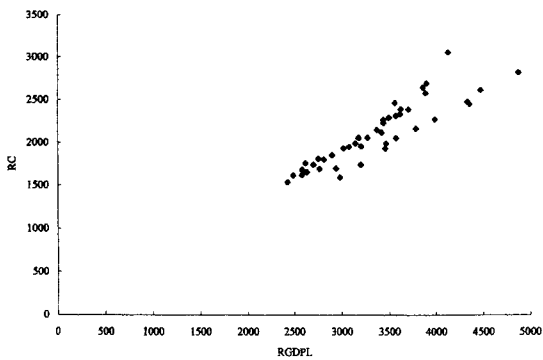


Fig. 7
(in level)

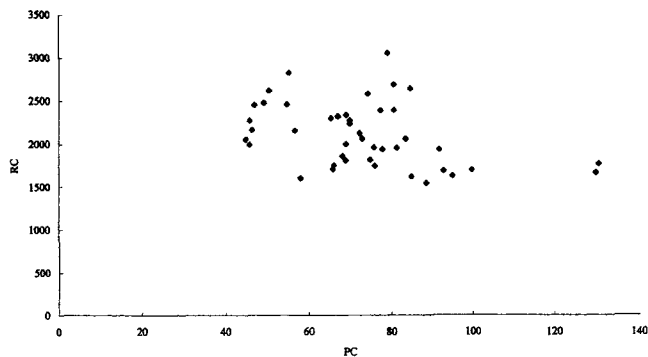


Fig. 8
(in level)

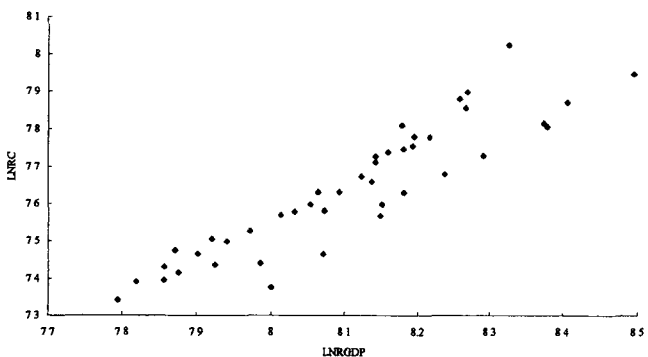


Fig. 9
(in logarithm)

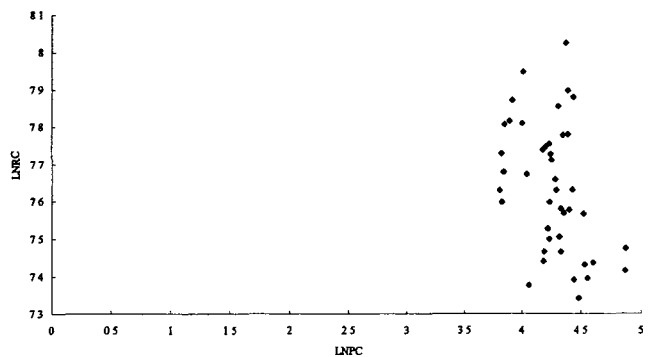


Fig. 10
(in logarithm)

Before moving on to the regression analysis, it would be helpful to report the summary statistics for the series, including their sample correlations and autocorrelations. Table 1 - Table 3 show the summary statistics, sample correlations and autocorrelations.³ Table 2 also supports the inference that the above mentioned two variables should be used for the explanation of the change in consumption

level, showing the strongly positive correlation between consumption and income as well as the negative correlation between consumption and consumption prices.

3 The existence of unit roots will be examined later. The unit roots in the AR model of consumption should not exist for the stability of the dynamic model studied in the next section.

Table 1 Sample period :1951 to 1992

Variable (s)	RC	RGDPL	PC
Maximum	305620.0	4886.0	130.3400
Minimum	159728.0	2487.0	44.6900
Mean	211966.8	3384.6	72.4507
Std. Deviation	36482.1	569.1548	19.3274
Skewness	.49050	.47848	1.0139
Kurtosis-3	-.48111	-.19233	1.7468
Coef of Variation	.17211	.16816	.26677
Variable (s)	LNRC	LNGDP	LNPC
Maximum	12.6301	:8.4941	4.8701
Minimum	11.9812	7.8188	3.7997
Mean	12.2501	8.1134	4.2502
Std. Deviation	.16929	.16614	.25727
Skewness	.20043	.12333	.13512
Kurtosis-3	-.87115	-.62072	.062359
Coef of Variation	.013820	.02477	.060530

LNRC=log (RC), LNGDP=log (RGDP), and LNPC=log (PC)

Table 2 Estimated Correlation Matrix of Variables
Sample period :1951 to 1992

RC	RC	RGDPL	PC
RGDPL	1.0000	.89655	-.37278
PC		1.0000	-.56924
LNRC	LNRC	LNGDP	LNPC
LNGDP	1.0000	.90880	-.38227
LNPC		1.0000	-.59478
			1.0000

Table 3 Sample period :1950 to 1992
Autocorrelation of LNRC

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic	Ljung-Box Statistic
1	.68592	.15250	20.2311 [.000]	21.6762 [.000]
2	.41829	.21246	27.7548 [.000]	29.9339 [.000]
3	.27203	.23082	30.9368 [.000]	33.5137 [.000]
4	.11594	.23816	31.5148 [.000]	34.1806 [.000]
5	.077970	.23947	31.7762 [.000]	34.4902 [.000]
6	.11038	.24006	32.3002 [.000]	35.1274 [.000]
7	.22479	.24123	34.4730 [.000]	37.8434 [.000]
8	.21629	.24606	36.4846 [.000]	40.4297 [.000]
9	.13572	.25044	37.2766 [.000]	41.4780 [.000]
10	.15005	.25214	38.2447 [.000]	42.7981 [.000]
11	.10355	.25421	38.7057 [.000]	43.4464 [.000]
12	-.0092567	.25519	38.7094 [.000]	43.4518 [.000]
13	-.098173	.25520	39.1238 [.000]	44.0734 [.000]
14	-.17066	.25608	40.3762 [.000]	46.0168 [.000]

Autocorrelation of LNGDP

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic	Ljung-Box Statistic
1	.83512	.15250	29.9893 [.000]	32.1314 [.000]
2	.67171	.23600	49.3908 [.000]	53.4257 [.000]
3	.53041	.27691	61.4881 [.000]	67.0352 [.000]
4	.41004	.29961	68.7179 [.000]	75.3773 [.000]
5	.33219	.31239	73.4630 [.000]	80.9965 [.000]
6	.29105	.32050	77.1056 [.000]	85.4267 [.000]
7	.27825	.32659	80.4348 [.000]	89.5881 [.000]
8	.26409	.33205	83.4337 [.000]	93.4439 [.000]
9	.22559	.33690	85.6219 [.000]	96.3400 [.000]
10	.18460	.34040	87.0873 [.000]	98.3383 [.000]
11	.13152	.34272	87.8310 [.000]	99.3842 [.000]
12	.029013	.34389	87.8672 [.000]	99.4367 [.000]
13	-.043598	.34395	87.9490 [.000]	99.5593 [.000]
14	-.094109	.34407	88.3298 [.000]	100.1503 [.000]

Autocorrelation of LNPC

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic	Ljung-Box Statistic
1	.83819	.15250	30.2100 [.000]	32.3679 [.000]
2	.59590	.23650	40.4792 [.000]	49.1267 [.000]
3	.39973	.26916	52.3500 [.000]	56.8564 [.000]
4	.25830	.28263	55.2190 [.000]	60.1668 [.000]
5	.13646	.28807	56.0198 [.000]	61.1150 [.000]
6	.048672	.28957	56.1216 [.000]	61.2389 [.000]
7	-.00855173	.28976	56.1247 [.000]	61.2428 [.000]
8	-.053229	.28977	56.2466 [.000]	61.3995 [.000]
9	-.034772	.28999	56.2986 [.000]	61.4683 [.000]
10	.053598	.29009	56.4221 [.000]	61.6367 [.000]
11	.083162	.29032	56.7195 [.000]	62.0549 [.000]
12	.017585	.29087	56.7328 [.000]	62.0742 [.000]
13	-.036742	.29090	56.7908 [.000]	62.1613 [.000]
14	-.023618	.29101	56.8148 [.000]	62.1985 [.000]

III Estimation of Dynamic Model by OLS

Using the above mentioned data, the following Autoregressive / Distributed Lag model has been estimated by the Ordinary Least Squares method⁴:

$$\begin{aligned}
 y_t = & \beta_1 + \beta_2 y_{t-1} + \beta_3 x_{1,t} + \beta_4 x_{1,t-1} \\
 & + \beta_5 x_{2,t} + \beta_6 x_{2,t-1} + u_t, \quad (1) \\
 u_t \sim & IN(0, \sigma^2), t = 1, \dots, T,
 \end{aligned}$$

where y_t represents real consumption per capita (LNRC) at time t , $x_{1,t}$ real GDP per capita (LNGDP) at time t , and $x_{2,t}$ con-

sumption prices (LNPC) at time t , respectively. All variables are expressed in logarithm. The estimation results are summarised in Table 4. As the table shows that the estimate of the coefficient of LNPC is not statistically significant⁵, the variable deletion

4 The assumption that the disturbance in (1) is a white noise guarantees that the estimator of parameters in (1) by OLS is consistent (Mann & Wald (1943)) under the assumption that the regularity conditions are satisfied, one of which will be examined later.

test, whose result is summarised in Table 5, has been conducted. The deletion of the variable, LNPC, has made the result better, and this deletion can be also supported statistically. In addition, the deletion of two variables, LNPC and LNPC(-1) has also been conducted. The deletion of all price variables from the model, however, can not be supported statistically. Hence, it can be concluded that the best estimate of the model,

which is shown in Table 5-1, is such that:

$$\begin{aligned} \text{LNRC}_t = & 1.1806 + .44331\text{LNRC}_{t-1} \\ & + 1.6537\text{LNGDP}_t - 1.0344\text{LNGDP}_{t-1} \\ & + .14153\text{LNPC}_{t-1} + \hat{u}_t, \end{aligned} \quad (2)$$

where \hat{u}_t denotes residuals. However, as shown in Table 5-1, it can be pointed out that there may be a problem of serial correlation in this estimation, which is indicated by Durbin's h-statistic and/or the langrange multiplier test of residual serial correlation

Table 4 Ordinary Least Squares Estimation

Dependent variable LNRC 42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	1.1792	.74659	1.5794 [.123]
LNRC (-1)	.43874	.13636	3.2175 [.003]
LNGDP	1.6547	.14460	11.4437 [.000]
LNGDP (-1)	-1.0296	.23025	-4.4715 [.000]
LNPC	.0095547	.060549	.15780 [.875]
LNPC (-1)	.13443	.062774	2.1414 [.039]
R-Squared	.92372	F-statistic F (5,36)	87.1830 [.000]
R-Bar-Squared	.91312	S.E. of Regression	.049900
Residual Sum of Squares	.089641	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	69.5465
DW-statistic	2.2972	Durbin's h-statistic	-2.0580 [.040]

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1) = 3.0667 [.080]	F (1, 35) = 2.7569 [.106]
B: Functional Form	CHI-SQ (1) = .75373 [.385]	F (1, 35) = .63959 [.429]
C: Normality	CHI-SQ (2) = .73450 [.693]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = .22778 [.633]	F (1, 40) = .21812 [.643]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

5 The probabilities shown in the brackets followed by T-Ratio in each table express the probabilities that the null hypothesis, $\beta_i = 0$, can not be rejected. Hence, the higher the number in the brackets becomes, the easier the null hypothesis can not be rejected.

6 Although several types of variables in level have been examined to estimate the coefficient in the model, equation (2) has been the best one.

in diagnostic tests. This implies that the estimator of parameters in this dynamic model is no longer consistent. In such a case another method such as the IV method must

be used to obtain a consistent estimator. However, it is also asserted that the small sample performance of the IV estimator will be poorer than that of the OLS estimator,

Table 5 Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of the variables deleted from the regression:			
LNPC			
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	1.1806	.73663	1.6027 [.117]
LNRC (-1)	.44331	.13148	3.3718 [.002]
LNGDP	1.6537	.14253	11.6020 [.000]
LNGDP (-1)	-1.0344	.22521	-4.5929 [.000]
LNPC (-1)	.14153	.043146	3.2803 [.002]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (1) = .029031 [.865]		
Likelihood Ratio Statistic	CHI-SQ (1) = .029041 [.865]		
F Statistic	F (1, 36) = .024901 [.857]		

Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of the variables deleted from the regression:			
LNPC LNPC (-1)			
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.0637	.76872	2.6846 [.011]
LNRC (-1)	.62394	.13385	4.6615 [.000]
LNGDP	1.5869	.15815	10.0336 [.000]
LNGDP (-1)	-1.2749	.23872	-5.3407 [.000]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (2) = 9.4853 [.009]		
Likelihood Ratio Statistic	CHI-SQ (2) = 10.7510 [.005]		
F Statistic	F (2, 36) = 5.2510 [.010]		

Table 5-1 Ordinary Least Squares Estimation

Dependent variable is LNRC			
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONTS	1.1806	.73663	1.6027 [.117]
LNRC (-1)	.44331	.13148	3.3718 [.002]
LNGDP	1.6537	.14253	11.6020 [.000]
LNGDP (-1)	-1.0344	.22521	-4.5929 [.000]
LNPC (-1)	.14153	.043146	3.2803 [.002]
R-Squared	.92366	F-statistic F (4, 37)	111.9222 [.000]
R-Bar-Squared	.91541	S.E. of Regression	.049238
Residual Sum of Squares	.089703	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	69.5319
DW-statistic	2.3043	Durbin's h-statistic	-1.8836 [.060]

even though the IV estimator is consistent. The estimation of the parameters in (1) obtained by the IV method is summarised in Table 6.

Furthermore, as mentioned before, the

sequences of economic variables should be stationary in order to make the classical regression model meaningful. The following section discusses how to investigate the stationarity of the data.

Table 5-1 Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)=3.0808 [.079]	F (1, 36)=2.8497 [.100]
B: Functional Form	CHI-SQ (1)=.82048 [.365]	F (1, 36)=.71728 [.403]
C: Normality	CHI-SQ (2)=.72647 [.695]	Not applicable
D: Heteroscedasticity	CHI-SQ (1)=.31425 [.575]	F (1, 40)=.30154 [.586]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

Table 6 Instrumental Variable Estimation

Dependent variable is LNRC				
List of instruments:				
CONST	LNGDP	LNGDP (-1)	LNGDP (-2)	LNPC (-1)
41 observations used for estimation from 1952 to 1992				
Regressor	Coefficient	Standard Error	T-Ratio [Prob]	
CONST	1.0760	.97789	1.1003 [.279]	
LNRC (-1)	.45953	.24264	1.8939 [.066]	
LNGDP	1.6604	.15825	10.4923 [.000]	
LNGDP (-1)	-1.0520	.36209	-2.9054 [.006]	
LNPC (-1)	.14017	.051906	2.7004 [.010]	
R-Squared	.91939	F-statistic F (4, 36)	102.6430 [.000]	
R-Bar-Squared	.91043	S.E. of Regression	.049836	
Residual Sum of Squares	.089410	Mean of Dependent Variable	12.2563	
S.D. of Dependent Variable	.16652	Value of IV Minimax	.0000	
DW-statistic	2.3491	Sargan's	NONE	

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)=.66792 [.414]	Not applicable
B: Functional Form	CHI-SQ (1)=.0042160 [.948]	Not applicable
C: Normality	CHI-SQ (2)=.67802 [.712]	Not applicable
D: Heteroscedasticity	CHI-SQ (1)=.71860 [.397]	Not applicable

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

IV Testing for Unit Roots

In order that equation (1) should be stable and the regularity condition for the argument of Mann & Wald (1943) should be satisfied, β_2 must be less than unity. This can be examined by conducting an ADF Test (Augmented Dickey-Fuller Test) in Microfit. The reported results are summarised in Table 7-1.

As Table 7-1 shows, the hypothesis that LNRC has unit roots can not be rejected. This result implies that equation (1) may neither be stable nor may the estimator of parameters by OLS be consistent, hence, suggesting that another method, such as the IV method, which produces consistent estimator (however, it will not generally be asymptotically efficient), or the two-step ML method, where it is possible to produce an estimator

which is consistent as well as asymptotically efficient, should be used.

For the testing of the static regression model in Section 5, it will be helpful to report several results on unit roots in the variables, LNGDP and LNPC. As the hypothesis testing of the static regression model implies the estimation of long-run equilibrium relationship, it may include the problem of co-integration, if all the variables have the same integrated order. The estimation results are shown in Table 7-2.

Table 7-2 shows that the hypothesis that there is a unit root can not be rejected for both variables. This means that all the variables used in this paper are not stationary. In order to check the integrated order of each variable, the ADF Test has been applied for the first difference of each variable. The results

Table 7-1 Unit root tests for variable LNRC

statistic	sample observations			without trend	with trend
	DF	1951	1992	42	-2.3954 (-2.9320)
ADF (1)	1952	1992	41	-2.4023 (-2.9339)	-3.3475 (-3.5217)
ADF (2)	1953	1992	40	-1.9982 (-2.9358)	-2.8894 (-3.5247)
ADF (3)	1954	1992	39	-2.1372 (-2.9378)	-3.2189 (-3.5279)

95% critical values in brackets.

Table 7-2 Unit root tests for variable LNGDP

statistic	sample observations			without trend	with trend
	DF	1951	1992	42	- .73627 (-2.9320)
ADF (1)	1952	1992	41	-1.0110 (-2.9339)	-2.7581 (-3.5217)
ADF (2)	1953	1992	40	- .76230 (-2.9358)	-2.5216 (-3.5247)
ADF (3)	1954	1992	39	- .45720 (-2.9378)	-2.4717 (-3.5279)

95% critical values in brackets.

Unit root tests for variable LNPC

statistic	sample observations			without trend	with trend
	DF	1951	1992	42	-1.7365 (-2.9320)
ADF (1)	1952	1992	41	-2.4450 (-2.9339)	-3.9280 (-3.5217)
ADF (2)	1953	1992	40	-1.9596 (-2.9358)	-3.1188 (-3.5247)
ADF (3)	1954	1992	39	-1.8885 (-2.9378)	-3.0646 (-3.5279)

95% critical values in brackets.

Table 7-3 Unit root tests for variable DLNRC

statistic	sample observations			without trend	with trend
	1952	1992	41		
DF				-6.5255 (-2.9339)	-6.4395 (-3.5217)
ADF (1)	1953	1992	40	-5.4940 (-2.9378)	-5.4149 (-3.5247)
ADF (2)	1954	1992	39	-3.9760 (-2.9378)	-3.9137 (-3.5279)
ADF (3)	1955	1992	38	-4.1970 (-2.9400)	-4.1225 (-3.5313)

95% critical values in brackets.

DLNRC denotes the first difference of LNRC

Unit root tests for variable DLNGDP

statistic	sample observations			without trend	with trend
	1952	1992	41		
DF				-4.9332 (-2.9339)	-4.8810 (-3.5217)
ADF (1)	1953	1992	40	-4.3038 (-2.9358)	-4.2600 (-3.5247)
ADF (2)	1954	1992	39	-3.6801 (-2.9378)	-3.6691 (-3.5279)
ADF (3)	1955	1992	38	-4.0290 (-2.9400)	-3.9796 (-3.5313)

95% critical values in brackets.

DLNGDP denotes the first difference of LNGDP.

Unit root tests for variable DLNPC

statistic	sample observations			without trend	with trend
	1952	1992	41		
DF				-4.7272 (-2.9339)	-4.6640 (-3.5217)
ADF (1)	1953	1992	40	-5.0117 (-2.9358)	-4.9264 (-3.5247)
ADF (2)	1954	1992	39	-4.1974 (-2.9378)	-4.1118 (-3.5279)
ADF (3)	1955	1992	38	-4.3921 (-2.9400)	-4.2676 (-3.5313)

95% critical values in brackets.

DLNPC denotes the first difference of LNPC

are in Table 7-3.

As each unit root test shows, the hypothesis on the existence of unit root can be rejected for all the first differences of variables. This implies that all variables have the same integrated order (i.e. $I(1)$), and the next step will be the estimation of co-integration vectors by Johansen Method, which can be used in Microfit. However, as this is obviously beyond the scope of this paper, further study has not been conducted.

V Testing for Restrictions and Model Selection

In this section, several types of restrictions have been examined. The restrictions examined in this paper are as follows:

1. Static Regression Model

$$\beta_2 = \beta_4 = \beta_6 = 0$$

2. Univariate Time Series Model

$$\beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$$

3. Leading Indicator Model

$$\beta_2 = \beta_3 = \beta_5 = 0$$

4. Finite Distributed Lags Model

$$\beta_2 = 0$$

5. Partial Adjustment Model

$$\beta_4 = \beta_6 = 0$$

6. Autoregressive Error Model

$$\beta_2\beta_3 + \beta_4 = 0, \text{ and } \beta_2\beta_5 + \beta_6 = 0$$

7. Dead-Start Model

$$\beta_3 = \beta_5 = 0$$

8. Single Explanatory Variable (No Inflation Effects)

$$\beta_3 = \beta_6 = 0$$

The results obtained for each restriction

are summarised in Tables 8-1 to 8-8. All tables show that all hypotheses for joint zero-restrictions can be rejected. This implies the estimation of all the restricted models corresponding to each of these hypotheses and the

examination of their diagnostic performances. The estimation of parameters and the diagnostic results are shown in Tables 9-1 to 9-8. The interesting results are clearly visible in Table 9-1, where the static regression

Table 8-1 (Static Regression Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of the variables deleted from the regression:			
LNRC (-1)	LNGDP (-1)	LNPC (-1)	
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.8472	.74618	3.8157 [.000]
LNGDP	1.0745	.074666	14.3907 [.000]
LNPC	.16117	.048218	3.3424 [.000]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (3)=18.3228 [.000]		
Likelihood Ratio Statistic	CHI-SQ (3)=24.0727 [.000]		
F Statistic	F (3, 36)= 9.2863 [.000]		

Table 8-2 (Univariate Time Series Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of the variables deleted from the regression:			
LNGDP	LNGDP (-1)	LNPC	LNPC (-1)
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	3.1274	1.2997	2.4062 [.021]
LNRC (-1)	.74558	.10621	7.0195 [.000]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (4)=34.8492 [.000]		
Likelihood Ratio Statistic	CHI-SQ (4)=74.3589 [.000]		
F Statistic	F (4, 36)=43.8614 [.000]		

Table 8-3 (Leading Indicator Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of variables deleted from the regression:			
LNRC (-1)	LNGDP	LNPC	
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	4.6725	1.2313	3.7949 [.001]
LNGDP (-1)	.89972	.12390	7.2617 [.000]
LNPC (-1)	.068698	.078204	.87846 [.385]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (3)=32.9509 [.000]		
Likelihood Ratio Statistic	CHI-SQ (3)=64.4701 [.000]		
F Statistic	F (3, 36)=43.6960 [.000]		

model has been examined, and in Table 9-8, where all price variables have been dropped from the model. When measured by adjusted R-squared, the performance of the model

which does not include any price variables is richer than that of the static model, although it is dubious that there are autocorrelated disturbance in the former model.

Table 8-4 (Finite Distributed Lags Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of variables deleted from the regression:			
LNRC (-1)			
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.4512	.70886	3.4579 [.001]
LNGDP	1.5248	.15541	9.8119 [.000]
LNGDP (-1)	-.42935	.15105	-2.8423 [.007]
LNPC	.050967	.066222	.76964 [.446]
LNPC (-1)	.16119	.069641	2.3147 [.026]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (1) = 9.3801 [.002]		
Likelihood Ratio Statistic	CHI-SQ (1) = 10.6153 [.001]		
F Statistic	F (1, 36) = 10.3520 [.003]		

Table 8-5 (Partial Adjustment Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC			
List of variables deleted from the regression:			
LNGDP (-1) LNRC (-1)			
42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	3.0189	.85046	3.5497 [.001]
LNRC (-1)	-.045065	.10324	-.43650 [.665]
LNGDP	1.1175	.12400	9.0120 [.000]
LNPC	.16852	.051554	3.2688 [.002]
Joint test of zero restrictions on the coefficient of deleted variables:			
Lagrange Multiplier Statistic	CHI-SQ (2) = 18.2041 [.000]		
Likelihood Ratio Statistic	CHI-SQ (2) = 23.8626 [.000]		
F Statistic	F (2, 36) = 13.7702 [.000]		

Table 8-6 (Autoregressive Error Model)
Wald test of restrictions imposed on parameters

Based on OLS regression of LNRC on:				
CONST	LNRC (-1)	LNGDP	LNGDP (-1)	LNPC
LNPC (-1)				
42 observations used for estimation from 1951 to 1992				
Coefficients A1 to A6 are assigned to the above regressors respectively				
List of imposed restriction (s) on parameter (s) :				
A2*A3+A4=0				
A2*A5+A6=0				
Wald Statistic	CHI-SQ (2) = 17.3410 [.000]			

Table 8-7 ("Dead-Start" Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC
List of variables deleted from the regression:
LNGDP LNRC
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	4.6928	1.4271	3.2883 [.002]
LNRC (-1)	-.0078074	.26690	-.029252 [.977]
LNGDP (-1)	.90833	.32005	2.8381 [.007]
LNGDP	.069993	.090747	.77130 [.445]

Joint test of zero restrictions on the coefficient of deleted variables:
Lagrange Multiplier Statistic CHI-SQ (2)=32.9507 [.000]
Likelihood Ratio Statistic CHI-SQ (2)=64.4691 [.000]
F Statistic F (2, 36)=65.5421 [.000]

Table 8-8 (Single Explanatory Model)
Variable Deletion Test (OLS case)

Dependent variable is LNRC
List of variables deleted from the regression:
LNRC LNRC (-1)
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.0637	.76872	2.6846 [.011]
LNRC (-1)	.62394	.13385	4.6615 [.000]
LNGDP	1.5869	.15815	10.0336 [.000]
LNGDP (-1)	-.2749	.23872	-5.3407 [.000]

Joint test of zero restrictions on the coefficient of deleted variables:
Lagrange Multiplier Statistic CHI-SQ (2)=9.4853 [.009]
Likelihood Ratio Statistic CHI-SQ (2)=10.7510 [.005]
F Statistic F (2, 36)=5.2510 [.010]

Table 9-1 (Static Regression Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC 43 observations used for estimation from 1950 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.8533	.71851	3.9710 [.000]
LNGDP	1.0738	.071288	15.0629 [.000]
LNPC	.16109	.047565	3.3867 [.002]
R-Squared	.87430	F-statistic F (2, 40)	139.1065 [.000]
R-Bar-Squared	.86801	S.E. of Regression	0.63051
Residual Sum of Squares	.15902	Mean of Dependent Variable	12.2430
S.D. of Dependent Variable	.17355	Maximum of Log-likelihood	59.3846
DW-statistic	1.5943		

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)=1.5911 [.207]	F (1, 39)=1.4985 [.228]
B: Functional Form	CHI-SQ (1)= .0059429 [.939]	F (1, 39)= .0053908 [.942]
C: Normality	CHI-SQ (2)=1.7529 [.416]	Not applicable
D: Heteroscedasticity	CHI-SQ (1)= .96248 [.327]	F (1, 41)= .93875 [.338]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

Table 9-2 (Univariate Time Series Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC 42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	3.1274	1.2997	2.4062 [.021]
LNRC (-1)	.74558	.10621	7.0195 [.000]
R-Squared	.55194	F-statistic F (1, 40)	49.2740 [.000]
R-Bar-Squared	.54074	S.E. of Regression	.11473
Residual Sum of Squares	.52651	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	32.3670
DW-statistic	1.8518	Durbin's h-statistic	.66183 [.508]

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)= .24439 [.621]	F (1, 39)= .22827 [.635]
B: Functional Form	CHI-SQ (1)= 5.0423 [.025]	F (1, 39)=5.3210 [.026]
C: Normality	CHI-SQ (2)=55.1575 [.000]	Not applicable
D: Heteroscedasticity	CHI-SQ (1)= 7.4873 [.006]	F (1, 40)=8.6778 [.005]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

Table 9-3 (Leading Indicator Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	4.6725	1.2313	3.7949 [.001]
LNGDP (-1)	.89972	.12390	7.2617 [.000]
LNPC (-1)	.068698	.078204	.87846 [.385]
R-Squared	.64594	F-statistic F (2, 39)	35.5749 [.000]
R-Bar-Squared	.62778	S.E. of Regression	.10329
Residual Sum of Squares	.41605	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	37.3114
DW-statistic	1.6282		

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1) = 1.5889 [.207]	F (1, 38) = 1.4941 [.229]
B: Functional Form	CHI-SQ (1) = .073552 [.786]	F (1, 38) = .066664 [.798]
C: Normality	CHI-SQ (2) = 30.2638 [.000]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = 3.6401 [.056]	F (1, 40) = 3.7957 [.058]

- A: Lagrange multiplier test of residual serial correlation
 B: Ramsey's RESET test using the square of the fitted values
 C: Based on a test of skewness and kurtosis of residuals
 D: Based on the regression of squared residuals on squared fitted values

Table 9-4 (Finite Distributed Lags Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.4512	.70886	3.4579 [.001]
LNGDP	1.5248	.15541	9.8119 [.000]
LNGDP (-1)	-.42935	.15105	-2.8423 [.007]
LNPC	.050967	.066222	.76964 [.446]
LNPC (-1)	.16119	.069641	2.3147 [.026]
R-Squared	.90178	F-statistic F (4, 37)	84.9253 [.000]
R-Bar-Squared	.89116	S.E. of Regression	.055852
Residual Sum of Squares	.11542	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	64.2388
DW-statistic	1.3516		

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1) = 4.7933 [.029]	F (1, 36) = 4.6378 [.038]
B: Functional Form	CHI-SQ (1) = .021286 [.884]	F (1, 36) = .018254 [.893]
C: Normality	CHI-SQ (2) = .50710 [.776]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = .45112 [.502]	F (1, 40) = .43430 [.514]

- A: Lagrange multiplier test of residual serial correlation
 B: Ramsey's RESET test using the square of the fitted values
 C: Based on a test of skewness and kurtosis of residuals
 D: Based on the regression of squared residuals on squared fitted values

Table 9-5 (Partial Adjustment Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC 42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	3.0189	.85046	3.5497 [.001]
LNRC (-1)	-.045065	.10324	-.43650 [.665]
LNGDP	1.1175	.12400	9.0120 [.000]
LNPC	.16852	.051554	3.2688 [.002]
R-Squared	.86536	F-statistic F (3, 38)	81.4087 [.000]
R-Bar-Squared	.85473	S.E. of Regression	.064526
Residual Sum of Squares	.15822	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	57.6151
DW-statistic	1.4975	Durbin's h-statistic	2.1911 [.028]

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)=6.1174 [.013]	F (1, 37)=6.3079 [.017]
B: Functional Form	CHI-SQ (1) = .047520 [.827]	F (1, 37) = .041910 [.839]
C: Normality	CHI-SQ (2) = .51516 [.773]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = .77850 [.378]	F (1, 40) = .75543 [.390]

- A: Lagrange multiplier test of residual serial correlation
 B: Ramsey's RESET test using the square of the fitted values
 C: Based on a test of skewness and kurtosis of residuals
 D: Based on the regression of squared residuals on squared fitted values

Table 9-6 (Autoregressive Error Model)
Non-Linear Least Squares Estimation
The estimation procedure converged after 3 iteration

Non-linear regression formula: LNRC=A1+A2*LNRC (-1)+A3*(1-A2)*LNGDP+A5*(1-A2)*LNPC (-1) 42 observations used for estimation from 1951 to 1992			
Parameter	Estimate	Standard Error	T-Ratio [Prob]
A1	2.4462	.84469	2.8960 [.006]
A2	-.044318	.095887	-.46219 [.647]
A3	1.1174	.073316	15.2413 [.000]
A5	.19732	.046600	4.2343 [.000]
R-Squared	.88014	F-statistic F (3, 38)	93.0128 [.000]
R-Bar-Squared	.87068	S.E. of Regression	.060880
Residual Sum of Squares	.14084	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	60.0577
DW-statistic	1.3426		

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1)=8.2913 [.004]	F (1, 37)=9.1008 [.005]
B: Functional Form	CHI-SQ (1) = .36046 [.548]	F (1, 37) = .32030 [.575]
C: Normality	CHI-SQ (2)=6.4898 [.039]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = .20147 [.654]	F (1, 40) = .19280 [.663]

- A: Lagrange multiplier test of residual serial correlation
 B: Ramsey's RESET test using the square of the fitted values
 C: Based on a test of skewness and kurtosis of residuals
 D: Based on the regression of squared residuals on squared fitted values

Table 9-7 ("Dead-Start" Model)
Ordinary Least Squares Estimation

Dependent variable is LNRC
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	4.6928	1.4271	3.2883 [.002]
LNRC (-1)	-.0078074	.26690	-.029252 [.977]
LNGDP (-1)	.90833	.32005	2.8381 [.007]
LNPC (-1)	.069993	.090747	.77130 [.0445]
R-Squared	.64594	F-statistic F (3, 38)	23.1093 [.000]
R-Bar-Squared	.61799	S.E. of Regression	.10464
Residual Sum of Squares	.41604	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	37.3119
DW-statistic	1.6230	Durbin's h-statistic	NONE

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1) = 6.2750 [.012]	F (1, 37) = 6.4989 [.015]
B: Functional Form	CHI-SQ (1) = .67786 [.795]	F (1, 37) = .059813 [.808]
C: Normality	CHI-SQ (2) = 29.7387 [.000]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = 3.6167 [.057]	F (1, 40) = 3.7686 [.059]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

Table 9-8 (Single Explanatory Variable)
Ordinary Least Squares Estimation

Dependent variable is LNRC
42 observations used for estimation from 1951 to 1992

Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	2.0637	.76872	2.6846 [.011]
LNRC (-1)	.62394	.13385	4.6615 [.000]
LNGDP	1.5869	.15815	10.0336 [.000]
LNGDP (-1)	-1.2749	.23872	-5.3407 [.000]
R-Squared	.90146	F-statistic F (3, 38)	115.8782 [.000]
R-Bar-Squared	.89368	S.E. of Regression	.055201
Residual Sum of Squares	.11579	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	64.1710
DW-statistic	2.2721	Durbin's h-statistic	-1.7723 [.076]

Diagnostic Tests

Test Statistics	LM Version	F Version
A: Serial Correlation	CHI-SQ (1) = 2.2864 [.131]	F (1, 37) = 2.1301 [.153]
B: Functional Form	CHI-SQ (1) = .89349 [.345]	F (1, 37) = .80423 [.376]
C: Normality	CHI-SQ (2) = .26092 [.878]	Not applicable
D: Heteroscedasticity	CHI-SQ (1) = .087411 [.767]	F (1, 40) = .083422 [.774]

A: Lagrange multiplier test of residual serial correlation

B: Ramsey's RESET test using the square of the fitted values

C: Based on a test of skewness and kurtosis of residuals

D: Based on the regression of squared residuals on squared fitted values

It is interesting to compare the results of these two models with that of the model discussed in Section 3, which is shown in Table 5-1. As there is doubt that autocorrelated disturbances may be found in both models -

the model with no dependent price variables and the model discussed in Section 3, namely equation (2) - it is reasonable to compare the result in Table 9-1 with the results in Table 10-1 and Table 10-2, where both models have

Table 10-1 (Single Explanatory Variable Model with AR Disturbance)
Exact AR(1) Inverse Interpolation Method Converged after 7 iterations

Dependent variable is LNRC 42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	1.0072	.62146	1.6207 [.113]
LNRC (-1)	.83056	.10962	7.5766 [.000]
LNGDP	1.6170	.13601	11.8891 [.000]
LNGDP (-1)	-1.4868	.20013	-7.4293 [.000]
R-Squared	.90915	F-statistic F (4, 37)	92.5669 [.000]
R-Bar-Squared	.89933	S.E. of Regression	.53715
Residual Sum of Squares	.10676	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	65.8046
DW-statistic	2.1657		

Parameters of the Autoregressive Error Specification

$$U = -.36743 * U(-1) + E$$

(-2.5603) [.015]

T-ratio(s) based on asymptotic standard errors in brackets

Log-likelihood ratio test of AR(1) versus OLS CHI-SQ(1)=3.2674 [.071]

Table 10-2 (Equation (2) in Section 3 with AR Disturbance)
Exact AR(1) Inverse Interpolation Method Converged after 7 iterations

Dependent variable is LNRC 42 observations used for estimation from 1951 to 1992			
Regressor	Coefficient	Standard Error	T-Ratio [Prob]
CONST	.43692	.56600	.77195 [.445]
LNRC (-1)	.65738	.11020	5.9655 [.000]
LNGDP	1.6972	.12154	13.9643 [.000]
LNGDP (-1)	-1.2945	.18526	-6.9875 [.000]
LNPC (-1)	.11294	.033843	3.3372 [.002]
R-Squared	.93061	F-statistic F (4, 37)	96.5679 [.000]
R-Bar-Squared	.92098	S.E. of Regression	.047590
Residual Sum of Squares	.081534	Mean of Dependent Variable	12.2501
S.D. of Dependent Variable	.16929	Maximum of Log-likelihood	71.4539
DW-statistic	2.1462		

Parameters of the Autoregressive Error Specification

$$U = -.39172 * U(-1) + E$$

(-2.7592) [.009]

T-ratio(s) based on asymptotic standard errors in brackets

Log-likelihood ratio test of AR(1) versus OLS CHI-SQ(1)=3.8439 [.050]

been re-estimated after having taken into account autocorrelated disturbances. The comparison of these three tables suggests that equation (2) with AR(1) disturbances should be used to estimate the relationship between consumption, income and consumption prices in Chile. Furthermore, the estimation of equation (2) by OLS in such a small sample can be supported by the argument that the small sample performance of the IV estimator will be poorer than that of the OLS estimator.

VI Conclusions

In this paper the economic relationship between consumption, production and consumption prices in Chile has been examined, using the data obtained from the Penn World Table. Examining several types of restrictions, it can be concluded that the best model for the study of the economic relationship among these variables, and the estimation of parameters of the model by the OLS is as follows:

$$LNRC_t = .43692 + .65738LNRC_{t-1}$$

$$+ 1.6972LNGDP_t - 1.2945LNGDP_{t-1} \\ + .11294LNPG_{t-1} + \hat{u}_t$$

where $LNRC$ denotes consumption, $LNGDP$ real GDP, and $LNPC$ consumption prices respectively, all of which are expressed in logarithm, and \hat{u}_t represents the residuals.

It should also be noted that the hypothesis that $LNRC$ has unit roots can not be rejected. This result implies that equation (1) may neither be stable nor may the estimator of parameters by OLS be consistent, hence, suggesting that another method, such as the IV method, which produces consistent estimator

(however, it will not generally be asymptotically efficient), or the two-step ML method, where it is possible to produce an estimator which is consistent as well as asymptotically efficient, should be used. The estimation result by the IV method is as follows:

$$LNRC_t = 1.0760 + .45953LNRC_{t-1} \\ + 1.6604LNGDP_t - 1.0520LNGDP_{t-1} \\ + .14017LNPC_{t-1} + \hat{u}_t$$

Furthermore, the following result has also been obtained: The hypothesis that there is a unit root can not be rejected for both $LNGDP$ and $LNPC$. This means that all the variables used in this paper are not stationary. On the other hands, the hypothesis on the existence of unit root can be rejected for all the first differences of variables. This implies that all variables have the same integrated order (i. e. $I(1)$), so that all variables are co-integrated. Hence, the next step, which has not been conducted in this paper, will be the estimation of the co-integrated vectors by the Johansen method and also the estimation of the long-run economic relationship in Chile by using ECM.

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