A DYNAMIC MODEL OF CLASSICAL SYNTHESIS*

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If we hope keynes' Principle of Effective Demand to be founded on a solid foundation, we should change the theory of value. We choose Sraffa's 1960 work or its modified generalization as the foundation of macrodynamic analysis, and study the problem on the determination of the rate of profits, following his ideas. Then, constructing a macrodynamic model as a subsystem of the whole, which will at least enable us to call prices into question, we analyse the dynamic process, while using R. M. May's researches on nonlinear difference equations. The economy grows, in general, along with historical time. We can consider that *the parameters characterizing the economy keep them from the chaotic region.*

All human activities are carried out in historical time. The present is the moment existing between the unchangeable past and the still unknown future, continuously moving, and man does all his work in this historical moment. The present state is the starting point for the future results from the products of the evolution of economic (and social) acts and is not independent from historical movements. The movements are essentially irreversible, and the economy evolves along with time.

In evolutional science, the analytical focus is brought to a point in which events evolve in a sequence of cause and effect, along with historical time. The general interest in non-equilibrium situations, in evolving systems of wide branches of science, has introduced important ideas in economics also, from the above mentioned essential properties of the economy. Among the economists closely related in their ideas on this subject, Alfred Marshall is worthy of special metion. He wrote:

> The Mecca of the economists lies in economic biology rather than in economic dynamics. But biological conceptions are more complex than those of mechanics; a volume on Foundations must therefore give a relatively large place to mechanical analogies; and frequent use is made of the term "equilibrium" which suggests

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> something of statical analogy. This fact, combined with the predominant attention paid in the present volume to the normal conditions of life in the modern age, has suggested the notion that its central idea is "statical", rather than "dynamical". But in fact it is concerned throughout with the forces that cause movement: and its key-note is that of dynamics, rather than statics.[1]

Concerning Marshall's ideal of economics, John Maynard Keynes took a strong revolutionary step against Marshall's theoretically inconsistent ideas. This revolution was made by describing economic activities on the basis of expectations and generally accepted habitual practices in irreversible historical time.[2] It was rightly interpreted by Joan Robinson.[3]

Recently, a group of economists represented by Kenneth J. Arrow, and a group of natural scientists who have developed techniques for studying nonlinear systems and adaptive paths in evolutionary systems, by bringing together two groups, explored the potential usefulness of a broadly transdisciplinary research program on the dynamics of the global economic system. They agreed that a more appropriate title for future research activities in this field would be "The Economy as an Evolving Complex System".[4] But the concept of General Competitive Equilibrium on which Arrow based his analyses was the one from which not only Keynes but also Marshall shook himself free when they studied the problem of economic progress, and did not reflect upon historical time. The whole analysis had been essentially carried out under the *ad hoc* and the unrealistic assumption that markets clear at every moment in the sense of general competitive equilibrium (or equilibrium dynamics). In other words, the time needed for arriving at the equilibrium has so far received totally inadequate consideration. One may then hope to get a better foundation upon which to build a sound Keynesian, or more precisely, non-Walrasian dynamic theory (Arrow [1988]; Grandmont [1985], pp.1039-1040).

This paper aims firstly to reconsider Keynes' Principle of Effective Demand. This principle is substantially in error on the aggregation of supply curves.[5] This problem has already been pointed out by piero Sraffa in his 1925 paper[6], long before the publication of *The General Theory*. It is essentially a problem of the theory of value. (I) Secondly, considering the price determination mechanism as the foundation of macrodynamic analysis, we finally choose Sraffa's 1960

1 *Principles of Economics*, Preface to the Eighth Edition, 1920.

2 *The General Theory of Employment Interest and Money*, 1936.

3 Presidential Adress, Section F, British Association, 1972. Reprinted in *After Keynes*, Basil Blackwell, 1973.

4 P. W. Anderson, K. J. Arrow and D.Pines (ed.), *The Economy as an Evolving Complex System*, the Proceedings of the Evolutionary Paths of the Global Economy Workshop, held September, 1987 in Santa Fe, New Mexico, Addison-Wesley Publishing Company, 1988.

5 *The Collected Writings of John Maynard Keynes* VII, *The General Theory*, p.385, Appendix I.

6 Sulle Relazioni fra Costo e Quantita Prodottà, *Annali di Economia*, vol.II, No.1, Milano 1925.

work[7] or its modified generalization. And we intend to show that "the rate of profits is susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest". (II) Thirdly, we will construct a macrodynamic model for an evolutional process analysis as a subsystem of the whole, which has been studied above, and analyze the dynamic economic process, while using Robert M. May's theoretical and experimental researches on nonlinear difference equation.[8] More concretely, we will use for this purpose L. L. Pasinetti's model[9] by replacing his linear consumption function with a nonlinear one as Keynes thought of it. Though this is the most important difference from his model, we consider coefficients which enter the equations to move along with historical time. Pasinetti's model is more general than those of P. A. Samuelson[10] or J. R. Hicks[11], and coincides with the one which has been adopted by J. S. Duesenberry[12], except for a few unimportant modifications. In our model, we show our analyses which are completely different from, and more realistic, than Pasinetti's. That is, the economy grows, in general, along with historical time. Of course, it may give rise to regular or irregular fluctuations around the growth trend by some exogenous factors, for example, monetary factors or technological progress, taken in conjunction with endogenous factors. For example, the firm's or consumer's behavior. We can consider that the parameters characterizing the economy keep them from the chaotic region. These results suggest the stability of the economic system. (III)

I Keynes' Principle of Effective Demand

Concerning the substantial character of the Principle of Effective Demand, it may be said that it is, in terms of traditional economics, a description of entrepreneurs' ordinary behaviour in which they determine a new level of activity upon the basis of a forecast of demand, taken in conjunction with the conditions of supply. In this entrepreneurs' activity, they naturally forecast the prices of goods at the time at which the goods will have been produced and be on sale. They look for maximizing expected profits, thinking about replacement of old and new equipment. It is of capital importance that the Principle of Effective Demand is, so to speak, the entrepreneurs' making of a desk plan; the amount of employment is determined at the intersection of aggregate demand and aggregate supply functions, which are conceived on the entrepreneurs' expectations of proceeds. Then they translate them into action. Keynes

7 *Production of Commodities by Means of Commodities*: Prelude to a Critique of Economic Theory, Cambridge.

8 Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles and Chaos, *Science* 186, pp.645~647, 1974;Simple Mathematical Models with Very Complicated Dynamics, *Nature* vol. 261, June 10,1976.

9 Cyclical fluctuations and economic growth, in his *Growth and Income Distribution*, Cambridge University Press, 1974.

10 Interaction between the Multiplier Analysis and the Principle of Acceleration, the *Review of Economics and Statistics*, 1939.

11 *A Contribution to the Theory of the Trade Cycle*, Oxford, 1950.

12 *Business Cycles and Economic Growth*, New York, 1958.

briefly explained as follows:

> The effective demand is simply the aggregate income (or proceeds) which the entrepreneurs expect to receive, inclusive of the incomes which they will hand on to the other factors of production, from the amount of current employment which they decide to give. The aggregate demand function relates various hypothetical quantities of employment to the proceeds which their output are expected to yield; and the effective demand is the point on the aggregate demand function which becomes effective because, taken in conjunction with the conditions of supply, it corresponds to the level of employment which maximises the entrepreneur's expectation of profit.

It is permissible for an entrepreneur to maximize the expectation of profit in the process of drawing up his desk plan. There would be no necessity for spending a lot of time to do so. But this entrepreneur's act is carried out at an historical time point. And "he has no choice but to be guided by these expectations, if he is to produce at all by processes which occupy time." That is, effective demand, and therefore output and employment are essentially dependent upon these expectations. Since this is so, there is no guarantee of being the optimum in reality (effects) even if he draws up the optimum as his desk plan (causes). The general competitive equilibrium theory envisions the effects to be optimum. Or, in other words, it supposes that the state of equilibrium is an approximation to reality. In doing so, it will be required to prove that we can estimate the real state at each historical time point as equilibrium, or necessary to prove that so much time is not needed to arrive at the equilibrium from the real state. There are none of these in the dynamics based on Walrasian equilibrium, though equilibrium dynamics theory certainly confirms these ideas. The equilibrium theory is a paradigm which optimizes behaviour of consumers and producers, but the optimization without thinking of historical time as the key to understanding how population and individuals survive runs the risk of confusing causes with effects.

On the other hand Keynes' Principle of Effective Demand is substantially in error on the aggregation of supply curves as written by the editors of *The Collected Writings* and has been discussed by several writers in *The Economic Journal* in the 1950's. There is no clear formulation in *The General Theory* about the determination of prices. Keynes wrote in the preface: "But our method of analysing the economic behaviour of the present under the influence of changing ideas about the future is one which depends on the interaction of supply and demand, and is in this way linked up with our fundamental theory of value. We are thus led to a more general theory, which includes the classical theory which are familiar, as a special case." Judging from this preface, the whole description in the book itself and recent research based on *The Collected Writings*, we could consider that the price determination mechanism in *The General Theory* did not break away from that of Marshall. That is, we could say that it was a marginal analysis, particularlly concerning decreasing returns (Asimakopulos [1991], Chapter 3).

Once the basic mechanisms of price determination in *The General Theory* have been stated as above, we can not ignore Sraffa's criticism that individual supply curves are not suitable except for the production conditions under which other things are equal. In order to add together individual supply curves for getting the supply curve of an industry as a whole, it is necessary to resort to a "trick" which substitutes industry's conditions for each producer's, thereby explaining the increasing cost. And if we hope the Principle of Effective Demand to be founded on a solid foundation, we should change the theory of value itself. This is required all the more from taking a modern view on price determination mechanism. That is, considering the existence of inventories, we cannot use directly the equations of demad and supply for determining prices as Marshall – and Walras – did (Hicks [1989], Chapter 1).

II Determination of the Rate of Profits in Sraffa's System

To begin with, we consider the uniform rate of profits in Sraffa's Production System as "the center of gravity" of different rates of profits for each industry. It may happen to be equal for some of them. Using the same notations in Sraffa's 1960 work, we write the equations of production in single-product industries:

$$(A_a p_a + B_a p_b + \cdots\cdots + K_a p_k)(1+r_a) + L_a w = A p_a$$

$$(A_b p_a + B_b p_b + \cdots\cdots + K_b p_k)(1+r_b) + L_b w = B p_b$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(A_k p_a + B_k p_b + \cdots\cdots + K_k p_k)(1+r_k) + L_k w = K p_k$$

Then,

$$[A-(A_a+A_b+\cdots+A_k)]\,p_a+[B-(B_a+B_b+\cdots+B_k)]\,p_b+\cdots+[K-(K_a+K_b+\cdots+K_k)]\,p_k$$
$$=[(A_a p_a+B_a p_b+\cdots+K_a p_k)\,r_a+(A_b p_a+B_b p_b+\cdots+K_b p_k)\,r_b+\cdots+(A_k p_a+\cdots+K_k p_k)\,r_k]+[L_a+L_b+\cdots+L_k]\,w.$$

Therefore, if the uniform rate r satisfies the following coudition:

$$(*)\quad [(A_a p_a+B_a p_b+\cdots+K_a p_k)\,r_a+(A_b p_a+B_b p_b+\cdots+K_b p_k)\,r_b+\cdots+(A_k p_a+B_k p_b+\cdots+K_k p_k)\,r_k]$$
$$=[(A_a p_a+B_a p_b+\cdots+K_a p_k)+(A_b p_a+B_b p_b+\cdots+K_b p_k)+\cdots+(A_k p_a+B_k p_b+\cdots+K_k p_k]\,r.$$

Then we obtain the same values of net national products and profits without distinguishing between using the uniform rate and the different rates of profits. We call the uniform rate r which satisfies the condition (*) "the center of gravity" of different rates $r_a, r_b, \cdots, r_k$. The above mentioned consideration does not need to be changed even in the system of multiple-product industries. As we will mainly study the movement of net national products (or income) along with time on the basis of Sraffa's theory of value and distribution, we consider, from the beginning, a uniform rate instead of different rates of profits.

In this paper I intend to show only that the rate of profits is susceptible of being determined from outside the system of production, in particular, by the level of the money rates of interest. We will not enter deeply into the filed of monetary theory, though recently there have been several interesting discussinos on the relation between the rate of profits (or the marginal efficiency of capital) and the money rates of interest, as noted by Sraffa and Keynes, in connection with F. A. von Hayek and J. G. K. Wicksell (Panico [1988]; Hishiyama [1993]).

Now, we shall construct a model of an economy, without abandoning the static point of view, but, at the same time, bringing ourselves as close as possible to the dynamic point of view. It will include the stationary state as a specific case. Here, we define the stationary state as the state in which "no changes in output and no changes in proportions in which different means of production are used by an industry are considered."

Suppose that our economy is made up of three sectors, whose balancesheet equilibrium has to be considered: the Firm Sector, the Household Sector, and the Public Sector. The Firm Sector produces all the commodities of basics and non-basics, and conducts real investment. The non-basic products have no part in the determination of values. Their role is purely passive. But in the situation which we are now considering, the non-basics are very important. The necessities of consumption and newly-invented machines are all contained in this class. The Household Sector consumes and saves.

For simplicity, we treat exogenously the Public Sector which is made up of financial institutions, government, and foreign trade.

Besides, we make the following assumptions:

(i) The supply of securities is limited to firms, and firms do not hold securities.

(ii) Only the Firm Sector holds real assets.

(iii) All the net profits of the Firm Sector revert to the Household Sector.

(iv) The Household Sector Supplies required labour at a certain money wage rate as far as the following analysis is concerned.

(v) The Exogenous Sector does not impose a tax upon the Firm or Household Sectors.

(vi) The Exogenous Sector expends the income from holding securities to purchase the labour services from the Household Sector.

With these simplifications in mind, we shall now formulate our model. But, some modifications of the system of production are required. With the exception of land (for simplicity), we have to include explicitly the non-basics in the whole system. We simply suppose that labour is measured in man-hours/time. Let us call N_1 amd N_2 the quantities of labour respectively employed in the basic and non-basic systems. Put $N\not{b} = N_1 + N_2$.

N_1, N_2 and therefore $N\not{b}$ is determined by the Principle of Effective Demand. In detail, we apply the Principle for each basic and non-basic industry, and then we sum up the whole quantities of labour to be employed in the basic or non-basic industries respectively. In this, we adopt the assumption of constant returns in all industries as a temporary working hypothesis, and we calculate the aggregate supply functions, using the aggreate demand functions in the ordinary way. The Principle of Effective Demand determines not only the amount of labour employed in each industry but also the scale of the whole system of production activity.

Next, we simply define the price-level (p) to be the price in terms of money of the composite commodity which is chosen as standard. We write this, symbolically, as follows:

$$[\text{MP of SCC}] \equiv p.$$

Obviously the price-level p is unknown. If the price-level and the rate of profits are determined from outside the system of production, then the determination of wage rate (W) and prices in terms of money will be deduced.

Our model of "outside the system of production" is prodused in Table I.[13] The suffix "t" stands for a time point. To avoid too complex notations, it is omitted without a minimum needed in the following discussion.

Table I
DEFINITIONS AND CLASSIFICATIONS OF VARIABLES

I. Endogenous variables

(a) Flow and Stock variables: (1) $Y_t (\equiv Y_S)$, Amount of production; (2) P_N, Net profits excluding the interest paid for securities; (3) I_t, Investment demand; (4) K_t, Capital stock; (5) Y_H, Household income; (6) C_t, Consumption demand; (7) S_H, Household savings

(b) Money market variables: (8) r, Rate of profits; (9) p, Price-level; (10) M_D^f, Demand for money by firms; (11) B_S^f, Supply of securities; (12) M_D^h, Demand for money by households; (13) B_D^h, Demand for securities by households

II. Initial conditions: M_O^f, Demand for money by firms; $\overline{V_O}$, Amount of real assets; $V_O \equiv \overline{V_O} + M_O^f$; B_O^h, Demand for seurities by households; A_O, Accumulative amount of household savings; $\overline{B}_o$, Securities held by Exogenous Sector

III. Other parameters: i, Basic money rate of interest determined by a central bank; G, Demand for products by Exogenous Sector; $\overline{B}$, Securities held by Exogenous Sector; N_D^a, Employment by Exogenous Sector;
$N_S^h \equiv N_D^f + N_D^a$, Total amount of labour employed

MODEL

Firm Sector:

(1) $Y_S \equiv iB_S^f + WN_D^f + P_N$

(2) $[\text{MP of SCC}] \equiv p$

(3) $K_t \equiv K_{t-1} + I_t$

(4) $I_t = \alpha Y_{t-1} - \beta K_{t-1} + A_2$

(5) $M_O^f + C_t + G + (B_D^h - B_O^h) + (\overline{B} - \overline{B_O}) = M_D^f + iB_S^f + WN_D^f + P_N$

(6) $M_D^f = M_D^f(i, r, V_O, Y_S)$

(7) $B_S^f = B_S^f(i, V_O, Y_S)$

Household Sector:

(8) $Y_H \equiv WN_S^h + iB_D^h + P_N$

(9) $C_t = A_1 + cY_{t-1} - bY_{t-1}^2$

(10) $Y_H \equiv C_t + S_H$

(11) $B_D^h + M_D^h \equiv A_O + S_H$

(12) $B_D^h = B_D^h(i, A_O, Y_H)$

Identity:[14]

(13) $Y_t (\equiv Y_S) \equiv C_t + I_t + G$

Here, A_1, A_2, c, b, α, β contained in equations (4) and (9) are all parameters which will be explained later in detail.

Equation (1) is the income statement of the firm sector. In identity (3), K_t represents the stock of capital at the end of the period "t" under consideration. It includes circulating or fixed capital which is calculated based

13 We owe much of our model to "*Nippon no Keikijunkan (Business Cycles in Japan)*, Tokyo 1965, Chapter 9" by Shozaburo Fujino. But there are essential differences between Fujino's model and ours. That is, Fujino's model is A MODEL OF NEO-CLASSICAL SYNTHESIS and ours is A DYNAMIC MODEL OF CLASSICAL SYNTHESIS.

upon the production system, and inventories.

The origin of function (4) is from thinking about investments as a function of the expected change in income. We make the assumption: the expected change in income that is normally used is the past change of income. And we suppose that when there is a discrepancy between desired and actual capital-output ratio, entrepreneurs may not carry out investment to cover the full difference, but only a fraction of it. Then we obtain the function (4) which is also called the "capital stock adjustment principle." And it is also considered as a generalization of the investment function used by M. Kalecki and N. Kaldor in the theory of business fluctuations. In general, $A_2 \geqq 0$, $\alpha \geqq 0$, and β may be either positive or negative. There could be a case in which the increase of capital equipment has a negative effect on investment. But capitalists would not cut down on investment if there was more demand than the normal production capacity, even in the same aspect. Therefore, we suppose β has both positive and negative values. We consider that A_2, α, β vary along with time.

Equation (5) is the balancesheet of the firm sector. We could consider it as the monetary support to carry out investment I_t.

Function (6) represents the money demand function of the firm sector. The amount of money demanded by a firm changes according to the total amount of initial assets and the amount of production which reflects the amount of money needed for transaction motive. And firms would decide the amount of money demanded by considering i and r. Funtction (7) is the supply function of securities of the firm sector which would depend on the total amount of initial assets V_0, and the amount of production Y_S, together with the money rate of interest i.

Indentity (8) is the definition of household income.

Function (9) stands for the total consumption as a nonlinear function of income. Keynes thought of a nonlinear consumption function with a negative 2-nd derivative from the beginning. It is clear that the nonlinear consumption function is more realistic than the linear one (Samuelson and Nordhaus [1992], pp.437-438). In the same way, as mentioned concerning the investment function, we consider that A_1, c, and b vary along with time. Here, A_1 is independent from the level of income, that is, a basic consumption. In general $A_1 > 0$, $c > 0$, and $0 < c - 2bY_t < 1$. And we consider $b > 0$ because "when real income increases, the community will wish to consume a gradually diminishing proportion of it."

Identity (10) is for the definition of household savings. And it stands for the

14 There are two other identities which are derived from our assumptions. In the first place, $P_N \equiv [(A_a p_a + B_a p_b + \cdots + K_a p_k) + \cdots + (A_k p_a + B_k p_b + \cdots + K_k p_k)]\, r$ + [Profits of non-basic sector]. Secondly, $B_s^f \equiv B_D^h + \overline{B}$. In our model, the basic money rate of interest is determined by a central bank. And we cannot consider that the money rate of interest carries out the function of adjusting supply and demand of the security market. Therefore, out of firm's supply of securities B_s^f, it becomes effective only by what is fulfilled by the household and exogenous sectors' demand [we have made the assumption: firms do not hold securities (i)]. And we could consider that B_s^f is identically equal to $B_D^h + \overline{B}$. Then, we obtain: $Y_H = Y_S$ + (New Money Supply).

expenditures of the household sector. Identity (11) represents the balancesheet of the household sector. And we could consider it as the monetary support to carry out consumption C_t. The demand for securities by the household sector would be a function of the money interest rate i, accumulative amount of household savings A_0, and household income Y_H. In that way, we obtain function (12).

Formula (13) is an indentity. Keynes worte in *The General Theory* as follows:

> When employment increases, aggregate real income is increased. The psychology of the community is such that when aggregate real income is increased aggregate consumption is increased, but not so much as income. Hence employers would make a loss if the whole of the increased employment were to be devoted to satisfying the increased demand for immediate consumption. Thus, to justify any given amount of employment there must be an amount of current investment sufficient to absorb the excess of total output over what the community chooses to consume when employment is at the given level. For unless there is this amount of investment, the receipts of the entrepreneurs will be less than is required to induce them to offer the given amount of employment. It follows, therefore, that, given what we shall call the community's propensity to consume, the equilibrium level of employment, i. e. the level at which there is no inducement to employers as a whole either to expand or to contract employment, will depend on the amount of current investment. The amount of current investment will depend, in turn, on what we shall call the inducement to invest; and the inducement to invest will be found to depend on the relation between the schedule of the marginal efficiency of capital and the complex of rates of interest on loans of various maturities and risks.
>
> ....
>
> The amount of labour N which the entrepreneurs decide to employ depends on the sum (D) of *two* quantities, namely D_1, the amount which the community is expected to spend on consumption, and D_2, the amount which it is expected to devote to new investment. D is what we have called the *effective demand.*

Therefore, in our case, which includes the demand for products by the exogenous sector, $Y_t \equiv C_t + I_t + G$.

Let us consider a subsystem which is made up of four expressions (3), (4), (9) and (13). The subsystem contains four variables K_t, C_t, I_t and Y_t, and is of a purely endogenous character.

The remaining subsystem of the whole concerns mainly money market variables except for variables which are determined in the above subsystem. And we could consider that the rate of profits is determined in this subsystem. That is, the rate of profits is susceptible of being determind from outside the system of production, in particular by the basic money rate of interest determined by a central bank.

Here, like Keynes, we consider that the above system of equations is "of the 'causal type', or, as we may say, of the 'decomposable type', as opposed to a completely interdependent system of simultaneous equations, that is, we suppose that a very definite logical succession according to which the variables are determined in the system (even if some of them may form among themselves smaller interdependent subsystems)".[15] When consumers intend to consume and entrepreneurs project to invest, each price of consumer goods and investment goods, the general level of prices and also the general rate of profits (as the center of gravity) have already been determined. Considering these prices of goods, the general level of prices and the general rate of profits, they carry out new plans of consumption and investment. In this way, the demand conditions are determined. The supply conditions of individual products and of products as a whole are determined from technical conditins, the level of wages, the extent of unused capacity of plant and labour, and the state of markets and competition, by the principle of effective demand. New prices — both individual prices and the price-level —, and profits emerge as the result of these two factors.

So far, we have studied a method for solving the problem, "Determination of the Rates of Profits in Sraffa's System". It has been done by a synthesis of revised Sraffa's and Keyns' systems. Our model may be called "A Dynamic Model of Classical Synthesis" which will be at least enable us to call prices into question, beyond the Fixprice Method. (Hicks [1965], Chapter 11; [1985], Chapter 12)

15 Keynes, op. cit., Preface to the French Edition, pp.XXXiV-XXXV; Pasinetti, The economics of effective demand, in *Growth and Income Distribution*, p.44.

III Macrodynamics as an Evolutional Process Analysis

Let us begin an investigation of the dynamic behavior of total effective demand in an economic system. For simplicity, we shall use the following theoretical model, omitting the demand for products by the exogenous sector G:

$$K_{t+1} \equiv K_t + I_{t+1} \qquad (1')-(3)$$

$$Y_{t+1} \equiv C_{t+1} + I_{t+1} \qquad (2')-(13)$$

$$C_{t+1} = A_1 + cY_t - bY_t^2 \qquad (3')-(9)$$

$$I_{t+1} = \alpha Y_t - \beta K_t + A_2 \qquad (4')-(4)$$

By substituting (3') and (4') into (2'), and putting $A \equiv A_1 + A_2$, we obtain:

$$Y_{t+1} = (A - \beta K_t) + (c + \alpha) Y_t - bY_t^2. \qquad (5')$$

We suppose, by a linear transformation

$$y_t \equiv \lambda Y_t + \mu, \qquad (6')$$

the expression (5') is transformed into

$$y_{t+1} = ay_t (1 - y_t). \qquad (7')$$

The linear transformation (6') means the conversion of a unit to measure total effective demand.

Then,

$$Y_{t+1} = \frac{a\mu - a\mu^2 - \mu}{\lambda} + a(1 - 2\mu) Y_t - a\lambda Y_t^2. \qquad (8')$$

The following equations are obtained by comparing (5') with (8'):

$$(a - a\mu - 1) \frac{\mu}{\lambda} = A - \beta K_t,$$

$a(1-2\mu) = c+\alpha$,
$a\lambda = b$.

By solving these equations for a, λ, and μ, we obtain:

$$a = 1 \pm \sqrt{(c+\alpha-1)^2 + 4(A-\beta K_t)\,b}, \quad (9')$$

$$\lambda = \frac{b}{a} \quad (10')$$

$$\mu = \frac{1}{2} - \frac{c+\alpha}{2a} \quad (11')$$

Form (7'), which is illustrated in Fig.1, is the simplest nonlinear difference equation, and $y_{t+1}=0$ at $y_t=0$ and $y_t=1$.

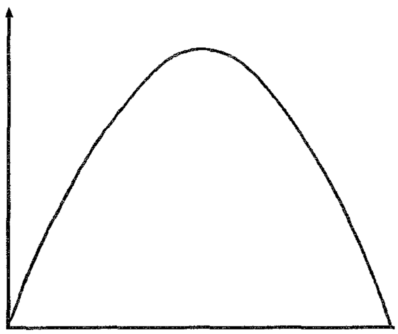


Fig. 1

For our applications, equation (7') requires y_t to remain on the interval $0 < y_t < 1$; if y_t ever exceeds unity, snbsequent iterations diverge towards $-\infty$ (which means total effective demand becomes nonexistent). Furthermore, y_t in equation (7') attains a maximum value of $\frac{a}{4}$ (at $y_t = \frac{1}{2}$) ; the equation therefore possesses non-trivial dynamical behavior only if $a<4$.

On the other hand, all trajectories are attracted to $y_t=0$ if $a \leqq 1$. (Fig.2) Thus for non-trivial dynamical behaviour, we require $1 < a < 4$; failing this, total effective demand becomes nonexistent. Thus, we consider only a which satisfies $1 < a < 4$ and the intial value y_0 which satisfies $0 < y_0 < 1$.

Then, from the assumption about coefficient b, $\lambda > 0$.

As for μ, if we take a number k : $0 < c+\alpha \leqq k$,

$$\mu = \frac{1}{2} - \frac{c+\alpha}{2a} \geqq \frac{1}{2} - \frac{k}{2a} = \frac{1}{2}\,(1-\frac{k}{a}).$$

Therefore, $\mu \geqq 0$ when $a \geqq k$.

On the nonlinear difference equation of (7'), there have been detailed numerical and mathematical studies. The results of numerical experiments tried by May are as follows: as the value of parameter a changes from 1 to 4, the nonlinear difference equation (7') can exhibit a remarkable spectrum of dynamic behavior. That is:

(i) $1 < a \leqq 2$. For any y_0, y_t is monotonically increasing and $y_t \rightarrow 1-(\frac{1}{a})$ as $t \rightarrow \infty$. (Fig.3)

(ii) $2 < a \leqq 3$. This case is essentially the same as the case (i).
For any y_0, $y_t \rightarrow 1-(\frac{1}{a})$ as $t \rightarrow \infty$. However, y_t moves not monotonically, but with damped oscillation after reaching a value over $1-(\frac{1}{a})$. (Fig.4)

(iii) $3 < a \leqq 1+\sqrt{6} = 3.449$. In this range, y_t approaches asymptotically a stable period 2 cycle. (Fig.5)

(iv) $1+\sqrt{6} < a$. The period 2 points will in turn become unstable and bifurcate to give an initially stable cycle of period 4. This in turn gives way to a cycle of period 8, and then

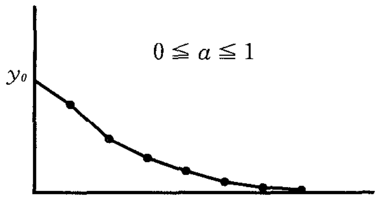


Fig. 2

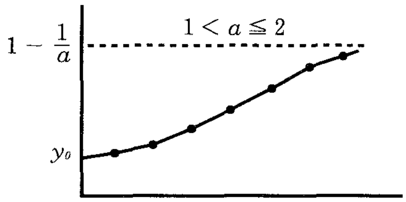


Fig. 3

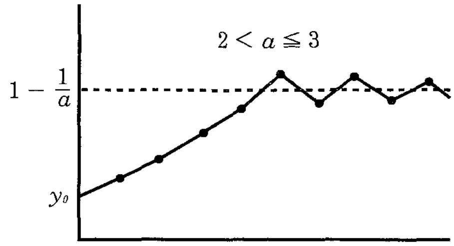


Fig. 4

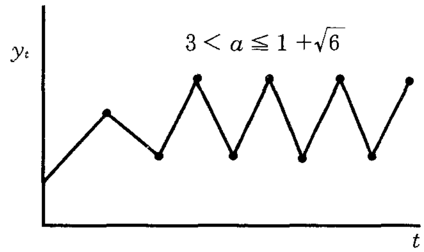


Fig. 5

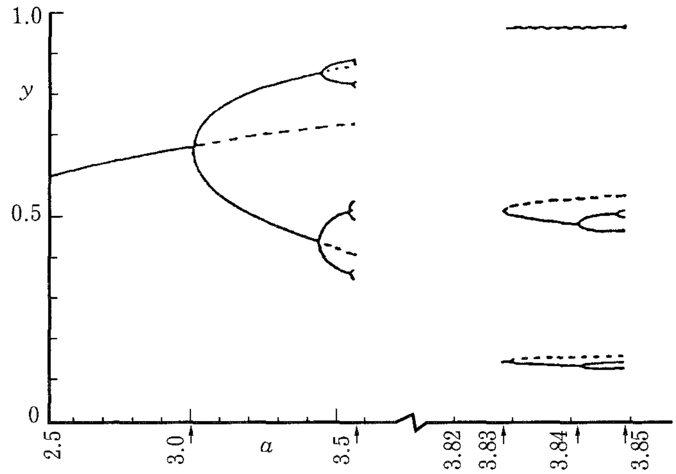
to a hierarchy of bifurcating stable cycles of periods 16, 32, 64, …………, 2^n.

Although this process produces an infinite sequence of cycles with period 2^n ($n \to \infty$), the "window" or range of parameter values wherein any one cycle is stable progressibly diminishes, so that the entire process is a convergent one, being bounded above by some critical parameter value. This critical parameter value is a point of accumulation of period 2^n cycles.

This limiting value of a has been tried to be calculated. It is known a approaches a_c = 3.57 ………. Beyond this point of accumulation there are an infinite number of fixed points with different periodicities, and an infinite number of different periodic cycles. There are also an uncountable number of initial points y_0 which give totally aperiodic (although bounded) trajectories; no matter how long the time series generated by (7') is extended, the pattern never repeats. These facts may be established by a variety of methods. Such a situation, where an infinite number of different orbits can occur, has been named "chaotic" by T－Y. Li and J. A. Yorke.[16] (Fig.6)

Now, we wish to call atteneion to the fact that the equation (5'), and therefore, the equations (7') and (8') are all higher order difference equations, having two variables Y_t or y_t, and K_t. And the greater part of May's numerical experiments and mathematical studies are concerned with first order difference equations. Nevertheless, there are some cases in which we can find the dynamics of solution on the basis of May's results. For example, in the case of (i), we could consider K_t as a parameter, and connecting relevant parts of the solution while shifting t period by period. Then, we obtain the dynamics of soulution in our case. (Fig.7) Similar remarks apply to the cases in which the coefficients in equations (3') and (4') vary along with time.

16 Period Three implies Chaos. *The American Mathematical Monthly* 82, 1975.



This figure illustrates some of the stable (—) and unstable (----) fixed points of various periods that can arise by bifurcation processes in equation (7'). To the left, the basic stable fixed point becomes unstable and gives rise by a succession of pitchfork bifurcations to stable harmonics of period 2^n; none of these cycles is stable beyond $a = 3.5700$. To the right, the two period 3 cycles appear by tangent bifurcation:one is initially unstable; the other is initially stable, but becomes unstable and gives way to stable harmonics of period 3×2^n, which have a point of accumulation at $a = 3.8495$. Note the cange in scale on the a axis, needs to be put on both examples on the same figure. There are infinitely many other such windows, based on cycles of higher periods. (Quoted from May's 1976 paper)

Fig. 6

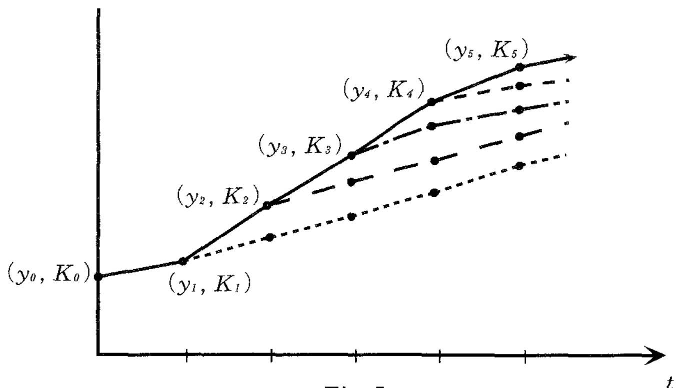


Fig. 7

As for the case of (ii), there are a variety of dynamics of solution. However, if they are restricted within an economically meaningful case, we could consider that the dynamics of solution increase at the initial time point.[17] And, we would obtain a similar result as mentioned in case (i).

Concerning other cases, the dynamics of

17 cf. R. L. Devaney, *An Introduction to Chaotic Dynamical Systems*, Addison-Wesley, New York, 1987, p.33, Proposition 5.3.

solution will be more complicated. We can find some descriptions of those cases in the final section entitled 'Related Phenomena in Higher Dimensions' of May's 1976 Paper. This field of study has been progressing most rapidly. (Jackson [1989, 1990])

The problem is "how large a value can a take in our model". Ilya Prigogine and Isabelle Stengers wrote in their "*Order out of Chaos*" as follows:

> But there is more. Instead of writing the logistic equation as continuous in time, let us compare the population at fixed time intervals (for example, separated by a year). This "discrete" logistic equation can be written in the form
>
> $$N_{t+1} = N_t\ (1 + r\ [1 - \frac{N_t}{K}]),$$
>
> where N_t and N_{t+1} are the populations separated by a one-year interval (we neglect here the death term). The remarkable feature, noted by R. May, is that such equations, in spite of their simplicity, admit a bewildering number of solutions. For value of the parameter $0 \leqq r \leqq 2$, we have, as in the continuous case, a uniform approch to equilibrium. For values of r lower than 2.444, a limit cycle sets in: we now have a periodic behavior with a two-year period. This is followed by four-, eight-, etc., year cycles, until the behavior can only be described as chaotic (if r is larger than 2.57). Here we have a transition to chaos as described in Chapter V. Does this chaos arise in nature? Recent studies seem to indicate that the parameters characterizing natural populations keep them from the chaotic region. Why is this so? Here we have one of the very interesting problems created by the confluence of evolutionary problems with the mathematics produced by computer simulation.[18]

Now, returning to our model, we will consider the same problem. From the relation which claims the marginal propensity to consume is positive, we obtain: $2bY_t < c$. Consequently,

$$Y_t < \frac{c}{2b} \quad (\text{for all } t). \tag{12'}$$

On the other hand, from the equation (5'),

$$A - \beta K_t = Y_{t+1} - (c + \alpha)\ Y_t + bY_t^2 < Y_{t+1} + bY_t^2.$$

If Y_t of the above inequality is replaced by $\frac{c}{2b}$, the following inequalities can be considerd:

$$A - \beta K_t < \frac{c}{2b} + b\ (\frac{c}{2b})^2 = \frac{c\ (2 + c)}{4b}.$$

Therefore,

$$4\ (A - \beta K_t)\ b < c\ (2 + c). \tag{13'}$$

Here, if we use $c = 0.9$, $\alpha = 0.5$, then we obtain:
$4\ (A - \beta K_t)\ b < 2.61$ from (13'), and hence from (9') :

$$a = 1 + \sqrt{(c + \alpha - 1)^2 + 4\ (A - \beta K_t)\ b} < 2.67. \tag{14'}$$

---

18 *Order out of Chaos, Man's New Dialogue with Nature*, Bantam Books, New York 1984, p.193.

In the same manner, we obtain:

$$\left.\begin{array}{l}\text{(I) if } c = 1.0,\ \alpha = 0.5, \\ \quad \text{then } a < 2.81 \\ \text{(II) if } c = 0.8,\ \alpha = 0.4, \\ \quad \text{then } a < 2.51\end{array}\right| \quad (15')$$

The parameters A ; c, b ; α , β and capital stock K may vary during constant creative destruction; that is economic evolution. We have to expect that during evolution, the values of those economic parameters will vary as well as many other parameters and values, whether they are quantifiable or not. The results (14') and (15') obtained above are no more than a standard, but we have chosen the values of c and α as comparatively larger and wider, according to factual findings. Therefore, from the above results, we could consider that the economy avoides the chaotic region as far as the movement of total effective demand is concerned.[19] Moreover, we could conclude a does not enter the region over 3. Here we write (9') again:

$$a = 1 \pm \sqrt{(c+\alpha-1)^2 + 4\,(A-\beta K_t)\,b}\,.$$

19 Even the Nonlinear Dynamics in Real-time Equity Market Indices, "There is clear evidence of nonlinear dependence at all frequencies,……. We find little to support the view that the process is chaotic at any frequency." (A. ABHYANKAR, L. S. COPELAND and W. WONG, Nonlinear Dynamics in Real-time Equity Market Indices: Evidence from the United Kingdom, IV. CONCLUSIONS, *The ECONOMIC JOURNAL*, VOL.105 NO.431, JULY 1995) It will be all the more difficult to support the view that the process is chaotic in the dynamic behavior of total effective demand in an economic system.

A change of the sign, from '−' to '+', or the reverse, in this formula, shows a change of growth patterns. That is, the former shows a change to the pattern of Fig.3 from that of Fig.2 and, vice versa. It seems likely to take a long time, or rather, a very long time. As compared with this, a change around a growth trend within a single growth pattern (for example, the pattern of Fig.3) is effected in a comparatively shorter time. Therefore, we could consider that a country's economy goes around a single growth trend, except for the special case in which a country moves to a new economic structure from a previous system, as in the case of the old East Germany or the old Soviet Union. Consequently, we could consider that a was always larger than 1. That is, we could say that our economy moves chiefly by following the pattern of dynamics of (i).

By solving the transformation (6') for Y_t,

$$Y_t = \frac{1}{\lambda}\,(y_t - \mu\,).$$

As mentined before, $\lambda > 0$. Then, if we consider that λ and μ move while keeping comparative stability along with time — the case in which all coeficients in equations (3') and (4') are constant is a special case —, the pattern of dynamics of y_t turns into that of Y_t on the whole. Of course, if some exogenous factors, (for example; the amount of bank loans and basic money rate of interest or technological progress,) taken in conjuntion with endogenous factors, (for example; the firm's or consumer's behavior) give rise to changes of λ and μ , then the changes invite regular or irregular movement around the growth trend. Historically, these changes have often been invited by way of stock markets. Furthermore, we have now known that the taxation system of a country has, like in America, a huge structural influence for the state of its economy (Barlet and Steele [1992]). Each historical aspect in the trade cycle, there were supplied with the peculiar conditions in that aspect. The evidence and facts show that, to go further on, there would be no other way except for examining peculiar features of each aspect, over our present scope.

REFERENCES

Arrow, K. J. (1988). Workshop on the Economy as an Evolving Complex System: Summary, in *The Economy as an Evolving Complex System* ed. by P. W. Anderson, K. J. Arrow and D. Pines, Addison-Wesley Publishing Company, pp.275-281.

Ashimakopulos, A. (1991). *Keynes's General Theory and Accumulation*, Cambridge University Press.

Barlett, D. L. and Steele, J. B. (1992). *America : What Went Wrong ?* Aitken & Stone Limited.

Grandmont, J.-M. (1985). On Endogenous Competitive Business Cycles, *Econometrica*, vol.53 No.5.

Hicks, J.(1965). *Capital and Growth*, The Clarendon Press, Oxford.

Hicks, J. (1985). *Methods of Dynamic Economics*, Oxford University Press.

Hicks, J. (1989). *A Market Theory of Money*, Oxford University Press.

Hishiyama, I. (1993). Sraffa-Riron no Gendaiteki Hyoka (The Modern Valuation of Sraffa's Theory), Kyoto University Press.

Jackson, E. A. (1989, 1990). *Perspectives of Nonlinear Dynamics*, Cambridge University Press, vo.1 (1989), vol.2 (1990).

Panico, C.(1988). *Interest and Profit in the Theories of Value and Distribution*, Macmillan Press.

Samuelson, P. A. and Nordhaus, W. D. (1992). *Economics*, 14th Edition, McGraw-Hill Inc..