Interest, Prices and a Critique of Economic Analysis

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We begin this paper by reconsidering about Sraffa’s well-known statement from the viewpoint of substantiation. And we develop our study for the foundations of “theory of interest, prices and a critique of economic analysis”. I think our study have great concern with present state of Japanese economy. Moreover, though in different stages each other, with present economic state of every country in the world.

Then, we consider some methodological problems on a critique of economic analysis.

I The Rate of Profits and Interest, the Case of Single-Product Industries and Circulating Capital

We suppose one sector consists of single industry producing a commodity. To begin with, Sraffa’s analyses are mainly devoted to statics. We aim to improve his model for dynamic analyses. For this reason, different from Sraffa’s model, we do not suppose the uniform rate of profits for all industries. The profits for which an industry finally aims to increase are the net profits deducted the costs for capital requisition. So, naturally, the rate of interests becomes another determination factor.

“The rate of profits, as a ratio, has a significance which is independent of any prices, and can well be ‘given’ before the prices are fixed. It is accordingly susceptible of being determined from outside the system of production, in particular by the level of the money rates of interest.”

(Sraffa [1960], p.33)
II The Case of Multiple-Product Industries and Fixed Capital

We do not have to change the above statement, excluding the wards ‘single industry producing a commodity’ from the case of single-product industries and circulating capital, for the case of multiple-product industries and fixed capital. Because “the rate of profits has a significance which is independent of any prices, and can well be ‘given’ before the prices are fixed”.

III Not ‘Switch in Methods of Production’, but Dynamics

1 The Model

Let us begin an investigation of the dynamic behavior of total effective demand in an economic system. For simplicity, we shall use the following theoretical model, omitting the demand for products by the exogenous sector:

\[
\begin{align*}
K_{t+1} &\equiv K_t + I_{t+1} \quad (1) \\
Y_{t+1} &\equiv C_{t+1} + I_{t+1} \quad (2) \\
C_{t+1} &\equiv A_1 + cY_t - bY_t^2 \quad (3) \\
I_{t+1} &\equiv \alpha Y_t - \beta K_t + A_2 \quad (4)
\end{align*}
\]

where: \( I = \) total net investment, \( C = \) total consumption, \( Y = \) total effective demand, or total net income, \( K = \) stock of capital at the end of the period under consideration, \( t = \) period of time which the variables refer to. The variable \( K \) represents a stock at the end of period \( t \); all other variables express flows which take place during period \( t \). The system \((1) - (4)\) is made up of four equations in four variables and of a purely endogenous character. And yet, this theoretical model is the same as Pasinetti’s ([1974], p.56), excluding that our consumption function \((3)\) is non-
linear and, more important from theoretical and positive view point also, we suppose $c$, $b$, $\alpha$, $\beta$, $A_1$, $A_2$ are able to change with time. Granted that we are permitted to suppose them constants in a certain time-interval containing a historical time point, it is reasonable to think that they change with time within longer interval. They certainly change with time as satisfying certain conditions that will be said later. An economy evolves and grows with historical time.

The formula (2) need not be explained. On (1), (3), and (4), it may be asked to do a certain extent.

About (1) first. The reason to be asked some explanation to this formula is that, the term ‘capital (stock)’ in its quantitative connotation have come to be inseparably linked with the supposition that they stand for quantities that can be measured independently of, and prior to, the determination of the prices of the products, as Sraffa said (Sraffa [1960], p.9). Therefore, when we use the term, it is required to construct theoretical foundation so that we can reply to his criticism. To work on this problem, as I have done before (Kajita [1995]), it would be required to synthesize a total macroeconomic analysis and a microanalysis containing the price-determination of products. Fortunately, it is not necessary to measure ‘capital (stock)’ correctly for developing the following argument.

Secondly, on (3). Formula (3) represents total consumption as a non-linear function of income $Y$. The concept of consumption function was introduced by Keynes, and based on the hypothesis that there was a stable empirical relationship between consumption and income. He wrote in General Theory as follows: “Since we are here concerned in determining what sum will be spent on consumption when employment is at a given level, we should, strictly speaking, consider the function which relates the former quantity ($C$) to the later ($N$). It is more convenient,
however, to work in terms of a slightly different function, namely, the function which relates the consumption in wage-units \((C_w)\) to the income in terms of wage-units \((Y_w)\) corresponding to a level of employment \(N\). …… We will therefore define what we shall call the *propensity to consume* as the functional relationship \(\chi\) between \(Y_w\), a given level of income in terms of wage-units, and \(C_w\) the expenditure on consumption out of that level of income, so that \(C_w = \chi(Y_w)\) or \(C = W \cdot \chi(Y_w)\).” (Keynes [1973], p.90) And moreover, “The marginal propensity to consume is not constant for all levels of employment, and it is probable that there will be, as a rule, a tendency for it to diminish as employment increases; when real income increases, that is to say, the community will wish to consume a gradually diminishing proportion of it” (op. cit., p.120) “As wealth increases \(dC/dY\) diminishes, but \(C/Y\) also diminishes. Thus the fraction increases or diminishes according as consumption increases or diminishes in a smaller or greater proportion than income.” (op. cit. p.126) All that Keynes wrote on the property of consumption function were as mentioned above.

We can find a concept of consumption function close to Keynes’ in Samuelson and Nordhaus’ *Economics*: “One of the most important relationships in all macroeconomics is the consumption function. The consumption function shows the relationship between the level of consumption expenditures and the level of disposable personal income (the under line is the writer’s). This concept introduced by Keynes, is based on the hypothesis that there is a stable empirical relationship between consumption and income. ……The relation between consumption and income shown in Figure 1 is called the consumption function.” (14th Ed, p.438) This concept relates personal consumption and income. On the other hand, Keynes’ concept concerns the nation as a whole.
Samuelson and Nordhaus have turned to a discussion of consumption for a nation as a whole, after they examined the budget patterns and consumption behavior of typical families at different income: “This transition from household behavior to national trends exemplifies the methodology of macroeconomics: we begin by examining economic activity on the individual level and then aggregate individuals to study the way the overall economy operates.” (op. cit., p.442) And, after a survey on the consumption behavior of society as a hole, they went on to say that: “Having reviewed the determinants of consumption, we may conclude that the level of disposable income is the primary determinant of the level of national consumption. Armed with this result, we can plot recent annual data on consumption and disposable income in Figure 2. The scatter diagram shows data for the period 1966-1991, with each point representing the level of consumption and income for a given year.” (op. cit., p.444) In addition, through the scatter points they have drawn a line - labeled CC and marked ‘Fitted consumption function’. The two concepts, the consumption function introduced by Keynes and the Fitted consumption function are completely different from each other. As explained before, the former is the functional relationship between income $Y$ and consumption $C$ in a single time period, and the latter is the trend of time series data over many periods.

Formula (3) stands for the consumption function of the short-term and the meaning of Keynes proper, by thinking about the consistency of formulae (1) – (4) as a whole. The stream of researches on consumer’s behavior in the history of macroeconomic analysis has flowed towards “Fitted consumption function from time-series data”. As we have seen, a linear approximation of consumption function has been used in general, after Keynes. For this, I cannot help to recollect Schumpeter’s words:
‘How seriously are we sinning if we decree that the function be linear?’ ([1954], p.1182)

Formula (3) is a curve of second order. The reason for this is that we have chosen the simplest one among which fulfill required properties. There are doubtless many higher order curves which partly have the properties. But those have them only partly and in effect, need the transformation of coordinates. As for which become the beginning of economic theses, it is thoughts and is not formula. After that, it is necessary to take a mathematical form as a quantitative theory to make the idea can compare them with the phenomenon. Therefore we must choose the simplest one. From those mentioned above, it is clear that: in general, \( A_1 > 0, \ c > 0, \ 0 < c - 2bY_t < 1 \). And we suppose \( b > 0 \), because “when real income increases, the community will wish to consume a gradually diminishing proportion of it.” (Keynes [1973], p.120)

Finally, on (4). This formula stands for the investment function known as ‘capital stock adjustment principle’. \( A_2 \) represents a spontaneous investment. In general, \( A_2 \geq 0, \ a \geq 0, \) and \( \beta \) stands for a positive or negative value. Moreover, we suppose these parameters are changeable with the passage of historical time, as in the case of consumption function.

2 Economic Growth with Evolution
Our model is the equation system (1) – (4). By substituting (3) and (4) into (2), and putting \( A = A_1 + A_2 \), we obtain:

\[
Y_{t+1} = (A - \beta K_t) + (c + a) Y_t - b Y_t^2.
\] (5)
We suppose, by a linear transformation

\[ y_t = \lambda Y_t + \mu, \]  

(6)

the expression (5) is transformed into

\[ y_{t+1} = ay_t (1 - y_t). \]  

(7)

The linear transformation (6) means the conversion of a unit to measure total effective demand.

Then,

\[ Y_{t+1} = \frac{a\mu - a\mu^2 - \mu}{\lambda} + a(1 - 2\mu)Y_t - a\lambda Y_t^2. \]  

(8)

The following equations are obtained by comparing (5) with (8):

\[ (a - a\mu - 1)\frac{\mu}{\lambda} = A - \beta K_t \]
\[ a(1 - 2\mu) = c + \alpha \]
\[ a\lambda = b \]
By solving these equations for $a$, $\lambda$, and $\mu$, we obtain:

\begin{align*}
  a &= 1 \pm \sqrt{(c + \alpha - 1)^2 + 4(A - \beta K_t) b} \quad (9) \\
  \lambda &= \frac{b}{a} \quad (10) \\
  \mu &= \frac{1}{2} - \frac{c + \alpha}{2a} \quad (11)
\end{align*}

Form (7), which is illustrated in Figure 3, is the simplest nonlinear difference equation, and $y_{t+1} = 0$ at $y_t = 0$ and $y_t = 1$. For our applications, equation (7) requires $y_t$ to remain on the interval $0 < y_t < 1$; if $y_t$ ever exceeds unity, subsequent iterations diverge towards $-\infty$ (that means total effective demand becomes nonexistent). Furthermore, $y_t$ in equation (7) attains a maximum value of $a/4$ (at $y_t = 1/2$); the equation therefore possesses non-trivial dynamical behavior only if $a < 4$.

Firstly, if $a \leq 1$, all trajectories are monotonically decreasing and attracted to $y_t = 0$.

Secondly, if $a > 3$, it may be possible to think that this case have not to do with real economy, by following reason. That is:

From the relation which claims the marginal propensity to consume is positive, we obtain: $2b Y_t < c$. Consequently,

\[ Y_t < \frac{c}{2b} \quad \text{(for all } t). \quad (12) \]

On the other hand, from the equation (5),

\[ A - \beta K_t = Y_{t+1} - (c + \alpha) Y_t + b Y_t^2 \\
< Y_{t+1} + b Y_t^2. \]

If $Y_t$ of the above inequality is replaced by $\frac{c}{2b}$, the following inequalities can be considered:
Therefore,

\[ 4(A - \beta K_t) b < c(2 + c). \]  \tag{13} 

Here, if we use \( c = 0.9 \), \( \alpha = 0.5 \), then we obtain:

\[ 4(A - \beta K_t) b < 2.61 \text{ from (13)}, \]

and hence from (9):

\[ a = 1 + \sqrt{(c + \alpha - 1)^2 + 4(A - \beta K_t) b} \]
\[ < 2.67. \]  \tag{14} 

In the same manner, we obtain \( a < 2.51 \), if \( c = 0.8 \), \( \alpha = 0.4 \). The meaning of these results is important. The parameters \( A \); \( c \), \( b \); \( \alpha \), \( \beta \) and capital stock \( K \) may vary during constant creative destruction; that is economic evolution. We have to expect that during evolution, the values of those economic parameters will vary as well as many other parameters and values, whether they are quantifiable or not. The results (14) and its supplement obtained above are no more than a standard, but we have chosen the values of \( c \) and \( \alpha \) as comparatively larger and wider, according to factual findings. Therefore, from the above results, we could consider that the economy keeps away from the chaotic region as far as the movement of total effective demand is concerned. If \( \alpha > 1 + \sqrt{6} \), the economy enters into the chaotic region. (Kajita [1995], pp.10-16); May [1976])

Following three cases have to do with actual economy:

(i) \( 0 < a \leq 1 \). For any \( y_0 \), \( y_t \) is monotonically decreasing and \( y_t \to 0 \) as \( t \to \infty \).

(ii) \( 1 < a \leq 2 \). For any \( y_0 \), \( y_t \) is monotonically increasing and \( y_t \to 1 - \)
\( \frac{1}{a} \) as \( t \to \infty \). (Figure 4)

(iii) \( 2 < a \leq 3 \). This case is essentially the same as the case (ii). For any \( y_0 \), \( y_t \to 1 - \frac{1}{a} \) as \( t \to \infty \). However, \( y_t \) moves not monotonically, but with damped oscillation after reaching a value over \( 1 - \left( \frac{1}{a} \right) \). (Figure 5)
The case: \(a > 1\). We wish to call attention to the fact that the equation (5), and therefore, the equation (7) and (8) are all higher order difference equations, having two variables \(Y_t\) or \(y_t\), and \(K_t\). Nevertheless, we can find the dynamics of solution on the basis of above examinations. For example, in the case of (ii) and (iii), we could consider \(K_t\) as a parameter, and connecting relevant parts of the solution while shifting \(t\) period by period. Then, we obtain the dynamics of solution in our case. (Figure 6) That is, \(y_t\) is monotonically increasing as time pass away.

【Similar remarks apply to the cases in which the coefficients in equations (3) and (4) vary along with time.】

By solving the transformation (6) for \(Y_t\),

\[Y_t = \frac{1}{\lambda} (y_t - \mu).\]

As mentioned before \(a > 1\), and \(b > 0\), therefore \(\lambda \left( = b/a \right) > 0\). And, from (11), \(\mu = 1/2 - (c + \alpha)/2a\). From the economical meaning of \(a, b, c, \alpha\), we are able to suppose, in usual situation, that \(\lambda\) and \(\mu\) move while keeping comparative stability along with time. In such a case, the pattern of dynamics of \(y_t\) turns into that of \(Y_t\) on the whole.

\([ II ]\) \(a \leq 1\). \(y_t\), consequently \(Y_t\) is monotonically decreasing in a restricted time interval. The case may be mere recession, or could be great depression as seen in the past.

It is properly to occur a transition from \([ I ]\) to \([ II ]\), or the reverse.

3 An Examination into the Historical Aspects of Japan-U.S. Economy

We have a favorable data on thinking about a relation between the above consideration and the dynamics of real economy (Shinohara, M. [1972, and 1987], United States Government Printing Office [2001]). It is in-
Interest to compare the above theoretical result and the facts findings.

IV A Critique of Economic Analysis

Notice that the above model and succeeding analyses are written by using difference equations. It is important to recognize big difference of results between by using continuous variables and differential equations or discontinuous variables and difference equations. To start with, on the case of using continuous variables and differential equations: the degree of abstraction is too high. Next, on the case of using discontinuous variables and difference equations: mathematical tools itself are now progressing (see, for example, E.A.Jackson, “Perspective of nonlinear dynamics, 1, 2”). Besides we throw some comments into the above; there is a third method: it is the method not using mathematics. This method is widely coming into use. Being made a wise selection for a problem, it is all right.

There is no alternative method consistently, and we make a selection of difference equations.

Reference


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