Summary of the Ph. D. Thesis

An Essay of Empirical Analysis of Capital Asset Pricing Model (CAPM)

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An Essay of Empirical Analysis of Asset Pricing Model

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The relationship between risk and return is the basic of the asset-pricing models. Since Elton (1999), some researchers argued on the use of alternative proxies for the expected return as they believe that average realized return cannot estimate the expected return, and fails to establish the relationship between risk and return. Although the researchers are more concerned about the alternative proxy for expected return (for example, Gebhardt et al. (2001), Brav et al. (2005), Easton and Monahan (2005), Pastor et al. (2008), Easton (2009), Lee et al. (2009), Guay et al. (2011) and Hou et al. (2012)), none of them argued on the reason behind the inability of realized return as the estimator for the expected return. Researchers implicitly assume realized returns as a sample of returns. And they mostly argued that as a sample, average realized return fails to estimate the expected return. This paper discusses that realized returns cannot be the sample of returns. Thus, if the realized return were not the ex-post realization of the ex-ante expectations, can we use average realized return to estimate the expected return?\(^1\)

Sharpe (1964), Lintner (1965) and Mossin (1966)’s Capital Asset Pricing Model (CAPM) is the first formal derivation of the risk-return relationship. CAPM established that in equilibrium, an asset’s expected return should be related to the associated risk, as measured by beta. To detect the positive risk-return relationship, empirical researchers have been using historical (realized) return data. In the text books, authors presented that the arithmetic mean of the realized returns

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\(^1\) In the empirical analysis of CAPM, researchers assume that the realized return is a sample of returns and the average of these returns will estimate the expected return. When the state of the future economy remains the same, the realized value and the ex-post value might coincide. Although, experimentally, we can assume same state of economy, empirically this would be an extreme case. The role of the asset-pricing is to determine the value of an asset by discounting the future expectations. If the economy remains the same, this would result in the same value for the asset over the years leaving the investors to invest into other opportunities.
and the sample variance can be treated as the estimates of the *ex-ante* parameters of returns, $\mu$ and $\sigma^2$ respectively.\(^2\) Thus the realized returns are assumed as a sample of returns.

**Fig 1: JSRI Tokyo 1st Section Market Index 1970-2008**

Following the textbook approach of testing the CAPM, we faced difficulties in explaining the negative average realized return for Tokyo Stock Exchange (TSE). Evidently, the monthly average realized market returns for TSE were negative for the samples 1990:1-1994:12 and 2000:1-2003:4. The average monthly realized market return for 1990-94 and 2000-03 were negative at -0.683% (-8.51% per year) and -1.71% (-22.56% per year), respectively. *Can the average realized return estimate the expected return for these samples of TSE when the average realized market return is negative?* If it is rational to conclude that the expected return would also be negative as the average realized market return is negative; *can we conclude that the* 

\(^2\)In empirical tests of the CAPM, it is believed that the average realized return will be a good estimator for the expected return. In testing the CAPM, researchers have been using realized return assuming that realized returns are normally distributed with mean $\mu_i$ and variance of $\sigma^2_i$: $R_i \sim N(\mu_i, \sigma^2_i)$, and over the sample period, the average of these realized returns will match the *ex-ante* expected return. Researchers believe, $\bar{\mu}_i \equiv \frac{1}{T}\sum_{t=1}^{T} R_i$ and $\bar{\sigma}_i^2 \equiv \frac{1}{T}\sum_{t=1}^{T}(R_i - \bar{R}_i)^2$. 
market was less risky during the period with negative average realized market return for TSE?³
Conversely, because of the economic bubble in Japan during 1985-89, the average monthly
realized market return for this period was as high as 2.097% (28.28% per year). Does this high
average realized return mean higher risk?⁴

Elton (1999) has concluded that the presence of information surprises hinders arithmetic
mean of the realized returns to be a good proxy for the expected return. He also argued on the
inability of the realized return to explain the risk-return relationship of the market. He noted ‘... in the recent past, the United States has had stock market returns of higher than 30 percent per
year while Asian markets have had negative return. Does anyone honestly believe that this was
the riskiest period in history for the United States and the safest for Asia?’ (p-1199). The
argument in Elton (1999) also implicitly assumed realized return as a sample of returns and this
inconsistency aroused as the realized return could not cancel out the information surprises. What
if the samples of realized returns in Elton (1999) did cancel out the information surprises? Could
we treat realized return as the estimator for expected return?⁵ Considering realized returns as
the ex-post realization of the ex-ante expectations has been misleading the researchers in

³ Japanese researchers might argue that this period was the adjustment period after the crash of JapaneseBubble during late 80s, and this sample should be discarded as the rational asset-pricing might be absent during this period. This negative average realized market return, for 1990:1~1994:12, along with the risk-free rate makes the risk-premium more negative; and this negative risk premium is contrary to the basic setting of CAPM in that it fails to explain positive linearity between risk and return. Thus any empirical tests using realized return as a sample would have led to the failure of CAPM, for the Tokyo Stock Exchange. To avoid controversies, researcher seldom discard these periods with negative average realized return or take longer sample to have a positive estimate of the expected return.

⁴ This sample period includes the economic bubble in Japan. The investors are least concerned about the inherent risk of their investment during an economic bubble. As for the investors, the enthusiasm of market participants, together with the inconsistent projection of fundamentals, contributed to a large degree to maintaining temporarily high asset prices during bubble. It was the excessive optimism rather than consistent projection of fundamentals that mainly supported temporarily high asset prices. Do these high returns mean the market was highly risky during this period? Can we explain the risk-return relationship by the observed prices during Japanese bubble? The use of realized returns to measure the risk-return relationship for this sample will be misleading.

⁵ Fama and French (fama-French Forum (May, 2009)) have argued that the risk-premium can be negative for a shorter sample period (for example, 10 years), and have argued on the rationale for considering longer sample period (35 years or more) to make the average realized return and the risk-premium positive. For 1927-2008 (82 years), the average annual difference between the market return (market portfolio of NYSE-Amex-NASDAQ stocks) and the return from rolling over one-month bills every month is 7.64% and the standard deviation (a statistical measure of volatility) of the annual differences is 21.04%. For four-year holding periods the average equity premium will be negative about 23% of the time. For 16-year holding periods, the probability of observing a negative average premium drops further, to about 7%. And for 25-year holding periods, the probability of a negative average premium is about 3.4%. Thus, even for quarter century holding periods, there is a 3.4% chance that bills will beat stocks.
empirical tests of the CAPM. Sharpe (1978), Black (1993) and Levy (2010) were concerned about ex-ante and ex-post values; yet none of them were able to derive the exact nature of the differences between these two values.⁶ We argue can realized return be the ex-post realization of the ex-ante return? Returning back to the origin of the asset-pricing model, we have redefined the ex-ante value, ex-post value and the realized value. As price is discounted value of next period’s expectations, we have shown that realized return cannot be a sample of return. We argue that considering realized returns as the sample of return is a fallacy that has misled the empirical researchers in estimating the expected return. We have shown that the price cannot be the ex-post realization of the ex-ante expectation.

The objective of this paper is to identify the reason behind the inability of the realized returns cannot be used as an estimator for the expected return. Researchers consider realized return as a sample of returns and have argued on the failure of the average realized return to estimate the expected return. Like other researchers we also believe that the average of the realized returns cannot estimate the expected return. However, we believe, the reason for this inability is that the realized return cannot be the sample of returns. Assuming realized returns as the ex-post realization of the ex-ante expectation is misleading, and this belief has led to empirical failure of the CAPM.

⁶ All the econometric sophistication in the world will not completely solve the basic problem associated with the use of ex post data to test theories dealing with ex ante predictions, however. The CAPM deals with predictions concerning a future period. It does not assume that the predictions or the implied relationships among them are stable over time. Nor does it assume that actual results will accord with such predictions, either period-by-period or, in any simple sense, “on average”. As the stock returns follow complex patterns, there are some factors that can have substantial impact for over longer periods of time (Sharpe (1978)). He concluded on the negative average return issue by saying, ‘unless one is prepared to reject the most rudimentary notion about risk aversion, it follows that at least some of the measured ex-post values do not adequately reflect ex-ante return predictions’.